High-Energy Astrophysics II

Assignment 2

Due: Thursday, May 8, 2014, 14:00
(Electronic submission to Markus.Bottcher@nwu.ac.za)

1) **On the nature of degenerate matter:** The Heisenberg Uncertainty Principle and the Pauli Exclusion Principle imply that the available phase space density of free electrons is

$$\frac{d^6 n_e}{d^3p \, d^3x} = \frac{2}{(2\pi \, \hbar)^3} \tag{2.1}$$

Now, degenerate matter (at 0 temperature) is defined by the condition that electrons are occupying all available momentum states up to a limiting value, called the Fermi momentum p_F, corresponding to a Fermi energy ϵ_F. In the non-relativistic case, we have $\epsilon_F = p_F^2/(2\,m_e)$. These limiting values are determined by the electron density through

$$n_e = \int\limits_0^{p_F} \frac{d^6 n_e}{d^3p \, d^3x} \, 4\pi \, p^2 \, dp. \tag{2.2}$$

a) Calculate the Fermi momentum p_F and the Fermi energy ϵ_F for non-relativistic degenerate electrons for a given density n_e.

b) Show that the average energy of electrons in a completely degenerate electron gas is given by $\overline{\epsilon} = (3/5)\,\epsilon_F$

c) Evaluate the pressure integral

$$P = \frac{1}{3} \int\limits_0^{p_F} pv \, n_e(p) \, dp \tag{2.3}$$

in the non-relativistic limit to show that the electron degeneracy pressure is given by

$$P_{\rm degen}^{NR} = \frac{(3\pi^2)^{2/3}}{5} \, \frac{\hbar^2}{m_e} \, n_e^{5/3}. \tag{2.4}$$

d) Set $v = c$ in Eq. (3) to show that the relativistic degeneracy pressure is given by

$$P_{\rm degen}^{R} = \frac{(3\pi^2)^{1/3}}{4} \, \hbar c \, n_e^{4/3}. \tag{2.5}$$

[30]

2) The calculation of the degeneracy pressure of neutrons is completely analogous to the case of electrons (see problem 2). Recall that the calculation of the Fermi momentum did not involve the particle mass at all.

   a) Find the expression for the neutron degeneracy pressure as a function of density ρ in the non-relativistic limit.

   b) Set this pressure value equal to the central pressure of the neutron star,

$$P_c = \frac{2}{3}\,\pi\,G\,\overline{\rho}_{\rm NS}^2\,R_{\rm NS}^2 \tag{3.1}$$

   to calculate the characteristic radius $R_{\rm NS}$ of a neutron star, as a function of its mass $M_{\rm NS}$.

[20]

3) Consider accretion onto a neutron star with mas $M = 1.4\,M_\odot$ and radius $r_* = 10$ km, resulting in a radiated luminosity of $L = 4 \times 10^{37}$ erg/s. The viscosity in the disk is parameterized through $\alpha = 0.1$, and the radial inflow velocity is $v_r = \beta\,c_s$, where $\beta = 10^{-2}$ and c_s is the sound speed.

   a) Find an expression for the stress $dF_\perp/dA$ on a vertical area element within the accretion flow, as a function of L, M, r_*, r, α, and β, and calculate the numerical value for the parameters listed above, at a radius of $r = 2\,r_*$.

   b) Estimate the magnetic field required to provide the stress found in part a), assuming that the magnetic-field pressure is a reasonable proxy for the stress mediated by magnetic fields.

[25]

4) Constraining Lorentz factors and viewing angles:

   a) Assume you observe both the approaching and the receding blob in an AGN jet from the same ejection event. At a given observing frequency, you observe fluxes F_a and F_r (in Jy) from the approaching and receding blob, respectively. The approaching blob exhibits superluminal motion with an apparent speed $\beta_{\perp,\rm app}$. Use the flux ratio F_a/F_r and the superluminal speed to calculate the bulk Lorentz factor Γ and the viewing angle θ. Assume that both blobs are emitting intrinsically the same spectrum with energy spectral index α, with the same co-moving intensity. You may neglect cosmological redshift effects.

   b) Now, assume we only see the approaching jet exhibiting superluminal motion at $\beta_{\perp,\rm app}$. However, we have an independent estimate of the Doppler factor δ, e.g., from brightness temperature arguments. Use these two quantities to calculate the Lorentz factor Γ and the viewing angle θ. You may neglect cosmological redshift effects.

[25]