Radiative Transfer

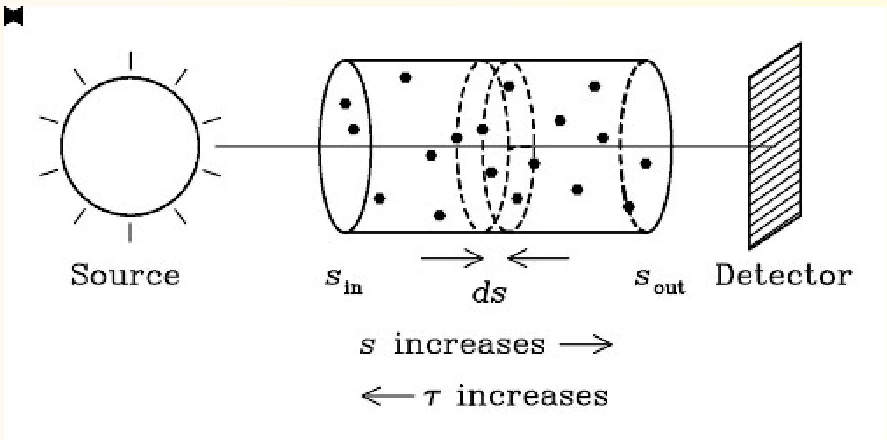
- Simple macroscopic model (no mechanism specified)
- Optical depth
- Absorption
- Emission
- Kirchoff's Law & equilibrium
- Atmospheric emission & absorption

In free space

- Along a ray path through vacuum, what goes in is what comes out
  - Intensity I_ν at frequency ν
  - along a ray path s from source (s=0) towards detector

$$\frac{dI_\nu}{ds} = 0$$

But if there is absorption



- optical depth τ (measured *away* from us)
- consider infinitesimal distance ds where probability of absorption at frequency ν is dp_ν
- define κ_ν as ratio

$$\frac{dp_\nu}{ds} \equiv \kappa_\nu$$

Absorption 1

- For Intensity I_ν at frequency ν

$$\frac{dI_\nu}{I_\nu} = -dp_\nu = -\kappa_\nu \, ds$$

$$\int_{s_{in}}^{s_{out}} \frac{dI_\nu}{I} = \ln[I_\nu(s_{out})] - \ln[I_\nu(s_{in})] = -\int_{s_{in}}^{s_{out}} \kappa_\nu(s) \, ds$$

$$I_\nu(s_{out}) = I_\nu(s_{in}) \times e^{-\int_{s_{in}}^{s_{out}} \kappa_\nu \, ds}$$

Absorption 2

define

$$\tau_\nu \equiv \int_{s_{in}}^{s_{out}} -\kappa_\nu(s)\,ds$$

$$I_\nu(s_{out}) = I_\nu(s_{in})\,e^{-\tau_\nu}$$

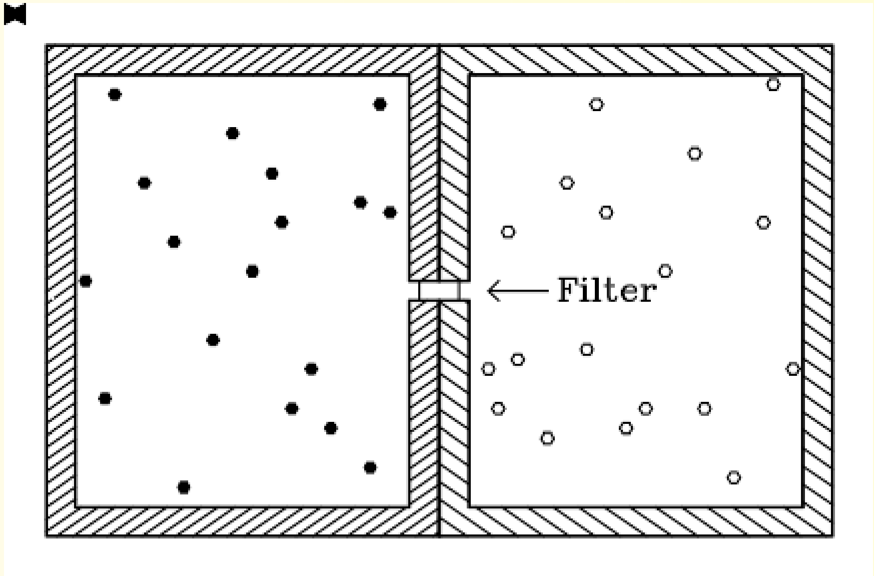
Emission along a path

- probability of emission from a volume (ds*dσ) per unit time into frequency ν into solid angle dΩ is propotional to (ds.dσ.dΩ) $\epsilon_\nu \equiv \frac{dI_\nu}{ds}$
- Does not depend on incoming emission (*except in non-equilibrium cases*)

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu$$

Kirchoff's Law
If it can absorb, it can emit

- Consider 2 cavities in thermodynamic equilibrium (same temperature)
- Filter between them allows through afreq range ν to $\nu+d\nu$



Black body equilibrium at temp T

- $I_\nu = B_\nu(T)$ blackbody radiation law
- no **net** change in passing through the filter

$$\frac{dI_\nu}{ds} = 0 = -\kappa_\nu B_\nu(T) + \epsilon_\nu$$

so ratio of emission to absorbtion is...

$$\frac{\epsilon_\nu(T)}{\kappa_\nu(T)} = B_\nu(T)$$

... the blackbody function

Equilibrium ?

- Only need a _local_ thermodynamic equilibrium (LTE) i.e some defined temperature
- Ratio κ/ε defined, but actual values depend on mechanism of emission or absoption
- If you can derive one, you have the other!

Atmosphere

- At ν >10GHz emission & absorption matter
- Pick a region of 'cold' sky (3K background), while atmosphere about 273K
- Using Rayleigh-Jeans approximation

$$I_\nu = \frac{2kT\nu^2}{c^2}$$

- **define** brightness temp $T_b(\nu)$ so that intensity is the same as that from the blackbody

$$T_b(\nu) \equiv \frac{I_\nu c^2}{2k\nu^2}$$

Tipping curve

- In practical use in radio astronomy
- Measure atmospheric emission to derive absorption
- Actual values depend on how much atmosphere you have above you (pressure), how hot it is, and the frequency you observe at

For our atmosphere at temp T_A

- Uniform atmosphere

$$-\kappa \, ds = d\tau \qquad \epsilon_\nu = \kappa_\nu B_\nu(T_A)$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu$$

$$\frac{1}{\kappa}\frac{dI_\nu}{ds} = \frac{-dI_\nu}{d\tau} = -I_\nu + B_\nu(T_A)$$

mulitply be $e^{-\tau}$ and integrate over τ

$$\int_0^{\tau_A} e^{-\tau} \frac{dI_\nu}{\tau} d\tau = \int_0^{\tau_A} [I_\nu - B_\nu(T_A)] e^{-\tau} d\tau$$

$$\int_0^{\tau_A} e^{-\tau} \frac{dI_\nu}{\tau} d\tau = [e^{-\tau} I_\nu]_0^{\tau_A} - \int_0^{\tau_A} -e^{-\tau} I_\nu d\tau = [e^{-\tau} I_\nu]_0^{\tau_A} + \int_0^{\tau_A} I_\nu e^{-\tau} d\tau$$

$$\int_0^{\tau_A} [I_\nu - B_\nu(T_A)] e^{-\tau} d\tau = \int_0^{\tau_A} I_\nu e^{-\tau} d\tau - B_\nu(T_A) \int_0^{\tau_A} e^{-\tau} d\tau$$

$$[I_\nu e^{-\tau}]_0^{\tau_A} = -B_\nu(T_A)(e^{-\tau_A} - 1)$$

more..

- from the last page

$$[e^{-\tau} I_\nu]_0^{\tau_A} = -B_\nu(T_A)(e^{-\tau_A} - 1)$$

- At the top of the atmosphere we are not looking at a strong source $I_\nu[\tau = \tau_A] \sim 0$

$$I_\nu[\tau = 0] = B_\nu(T_A)(1 - e^{-\tau_A})$$

finally!

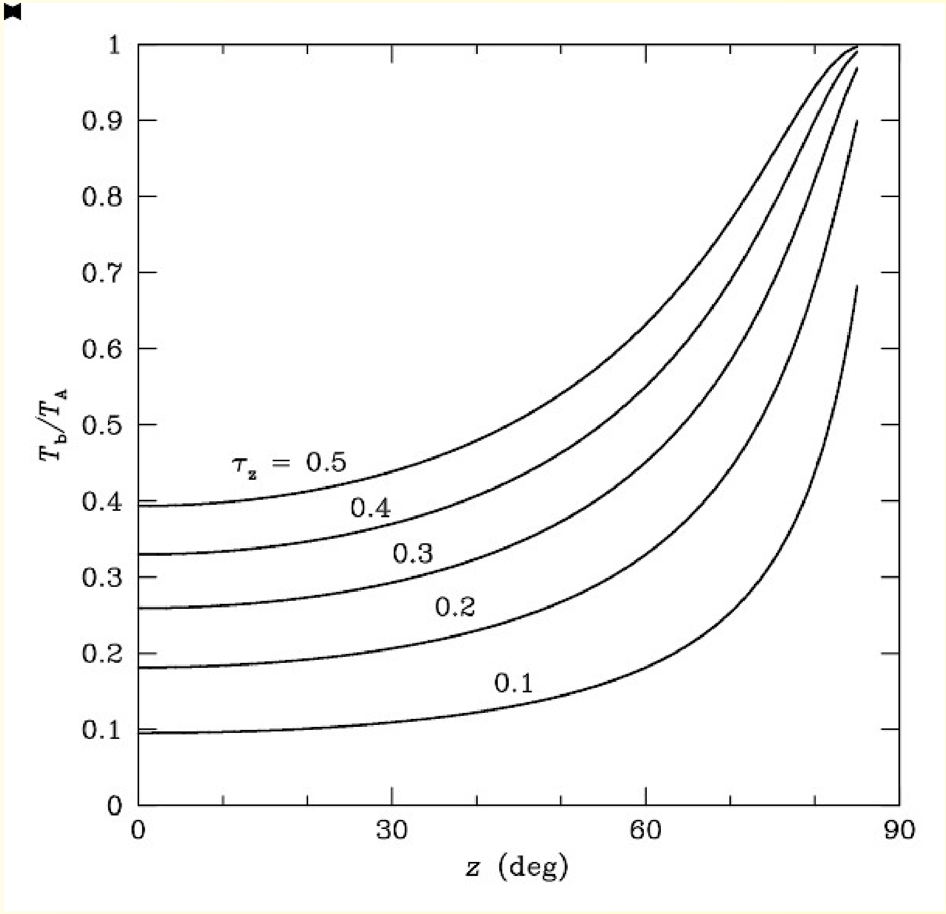
- If we assume a plane parallel uniform atmosphere at a zenith angle (z)

$$\tau_A = \tau_z \sec(z)$$

- For large zenith angles you must account for the curvature of the earth, refraction etc...

Example: GBT at 90GHz

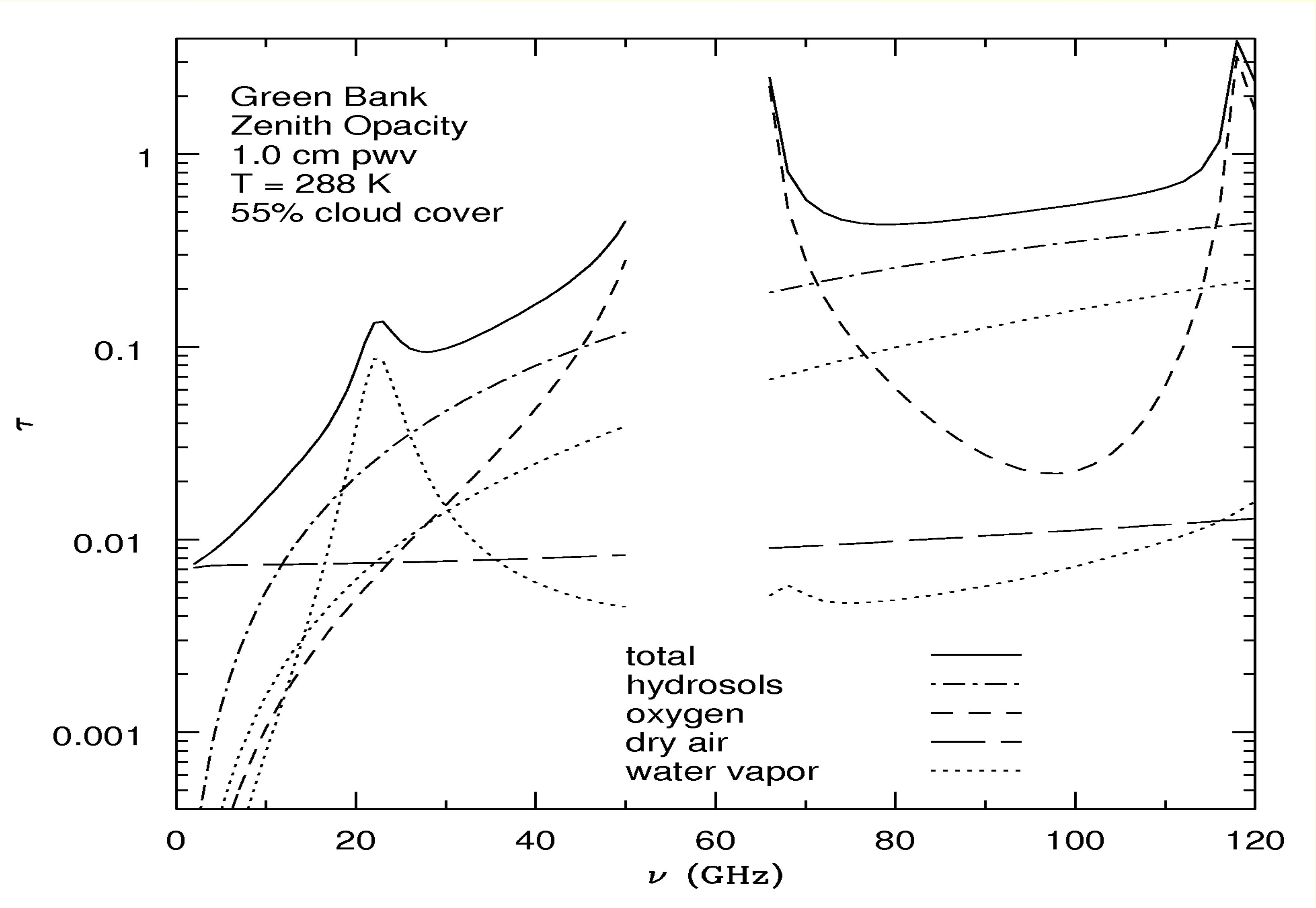
- cold weather (T=273K) τ_Z~0.1
- fit to get intensity *before* atmosphere



Optical depth values

- Does not depend on the receiver, or other equipment
- Depend on zenith angle (there are corrections for the plane parallel value at high z), i.e. how much atmosphere
- Varies a lot with frequency; at zenith about 1GHz maybe < 1%, can go to >10% at 30GHz, very high above 60GHz
- The atmospheric emission can dominate over the power from the radio source!

Green Bank Telescope (USA)



Reflector surface

- **Must not** absorb radio waves (otherwise it emits them at ~300K)
  - so paint must be transparent to radio
- **Should** scatter sunlight (do not want receiver to heat up!)
  - generally 'Goldstone' white
  - less of a problem in some places...
    - if the surface is a mesh
    - if it is very cloudy