Blackbody radiation

- Note: I am not following the derivation directly from the webite http://www.cv.nrao.edu/course/astr534/PDFnew.shtml
- It needs QM treatment to understand where it comes from (classical treatment has no limit to power you would emit). I use one from Harwit
  - *Martin Harwit 'Astrophysical Concepts'*
- You **do not need to repeat** the derivation, but do need to understand it!

6D Phase space

- In quantum mechanics particles are identical if they have the same spin and occupy the same cell in phase space (3D position, 3D momentum)

$$\delta x \, \delta y \, \delta z \, \delta p_x \, \delta p_y \, \delta p_z = h^3$$

- so if we allow up to a maximum momentum p_{max} and 2 spin states we have a max number of particles in a volume V:

$$\frac{8\pi p_{max}^3 V}{3 h^3}$$

Photons

- 2 spins (LHC, RHC) bosons
- Freq ν

$$\nu \equiv \frac{pc}{h}$$

- number of phase space cells from p to p+dp

$$Z(p)\,dp = 2\mathrm{V}\frac{4\,\pi\,p^2\,dp}{h^3}$$

- so from ν to ν+dν

$$Z(\nu) = 2\left[\frac{4\pi\nu^2\,d\nu}{c^3}\right]V$$

States & Probabilities

- Bosons can aggregate with zero point energy
- Relative probability for photon to be in the nth energy state at temperature T

$$e^{-((n+\frac{1}{2})h\nu)/kt}$$

- Absolute probability

$$\frac{e^{-(n+\frac{1}{2})h\nu/kt}}{\sum_{n}^{\infty} e^{-(n+\frac{1}{2})h\nu/kt}} = \frac{e^{-(nh\nu)/kt}}{\sum_{n}^{\infty} e^{-(nh\nu)/kt}}$$

Average energy per phase cell

- energy of that cell times its probability

$$\frac{\sum_{n} (n+\frac{1}{2}) h\nu e^{-nh\nu/kt}}{\sum_{n}^{\infty} e^{-nh\nu/kt}}$$

- replace hν/kT by x

$$\langle E \rangle = \frac{kT(xe^{-x} + 2xe^{-2x} + 3xe^{-3x} + ...)}{1 + e^{-x} + e^{-2x} + e^{-3x} + ...} + h\nu/2$$

continued..

- using (1+a+a^2 +a^3...) = 1/(1-a)
- numerator

$$kT[(x(e^{-x}+e^{-2x}+e^{-3x}+\ldots)+x(e^{-2x}+e^{-3x}+\ldots)+x(e^{-3x\ldots})+\ldots)]$$

$$kT[\frac{xe^{-x}}{1-e^{-x}}+\frac{xe^{-2x}}{1-e^{-x}}+\frac{xe^{-3x}}{1-e^{-x}}+\ldots]$$

$$\frac{kTxe^{-x}}{(1-e^{-x})^2}$$

- denominator

$$\frac{1}{(1-e^{-x})}$$

Finally

- combining back

$$\langle E \rangle = \frac{kTxe^{-x}}{1-e^{-x}} + \frac{h\nu}{2} = \frac{kTx}{e^{x}-1} + \frac{h\nu}{2} = \frac{h\nu}{e^{h\nu/kT}-1} + \frac{h\nu}{2}$$

- so energy density per phase cell times phase cells <u>per unit volume</u> gives

$$\rho(\nu, T)\, d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \left(\frac{h\nu}{e^{h\nu/kT}-1} + \frac{h\nu}{2} \right)$$

- we can ignore ½hυ as it is not observable in emission or absorption; it is a zero-point

Blackbody

- General form

$$\rho(\nu,T)d\nu=\frac{8\pi\nu^2 d\nu}{c^3}\left(\frac{h\nu}{e^{h\nu/kT}-1}\right)$$

- Intensity per unit area per unit frequency per unit solid angle (expansion at c over 4π steradians)

$$I(\nu,T)=\frac{c\rho(\nu,T)}{4\pi}=\frac{2\nu^2}{c^2}\left(\frac{h\nu}{e^{h\nu/kT}-1}\right)$$

Stefan's Law

- integrate to infinity

$$\int_0^{\infty} \frac{2h\nu^3}{c^2}\left(\frac{1}{e^{h\nu/kT}-1}\right)d\nu=\sigma T^4$$

- where σ is Stefan's constant, the radiation per unit area

$$\sigma=\frac{2\pi^5 k^4}{15 h^3 c^2}$$

$5.67\text{x}10^{-8}\,\text{Wm}^{-2}\,\text{K}^{-4}$

Approximations

- AT low frequency (radio) $e^{h\nu/kT} \sim 1 + h\nu/kT$

$$B(\nu,T) = \frac{c\,\rho(\nu,T)}{4\pi} = \frac{2\nu^2}{c^2}\left(\frac{h\nu}{e^{h\nu/kT}-1}\right) \simeq \frac{2kT\,\nu^2}{c^2}$$

- At high frequency (optical and above)

$$e^{h\nu/kT} >> 1$$

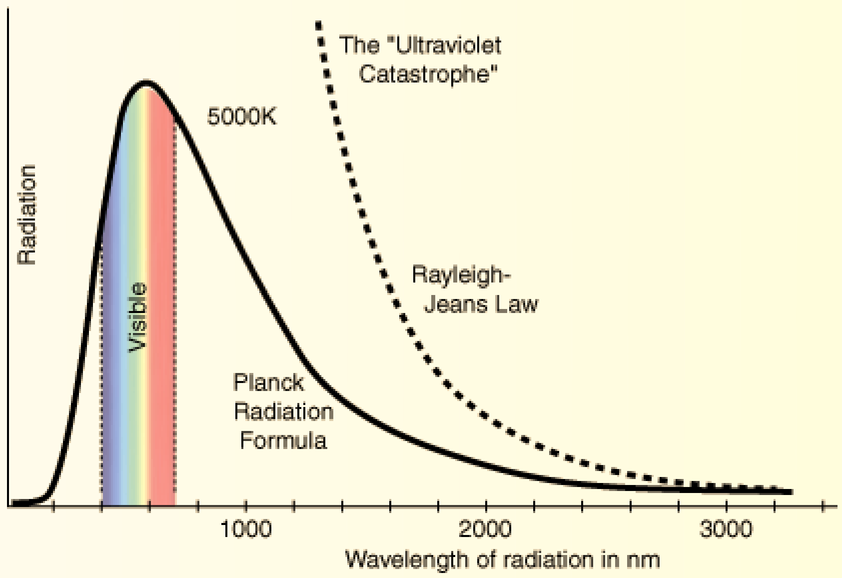
$$B(\nu,T) \simeq \frac{2h\,\nu^3}{c^2}\left(\frac{1}{e^{h\nu/kT}}\right)$$

Wien's Law

- By differentiation

$$\frac{\partial B}{\partial \nu}=0 \quad when \quad \nu=59\,GHz \times T(K)$$

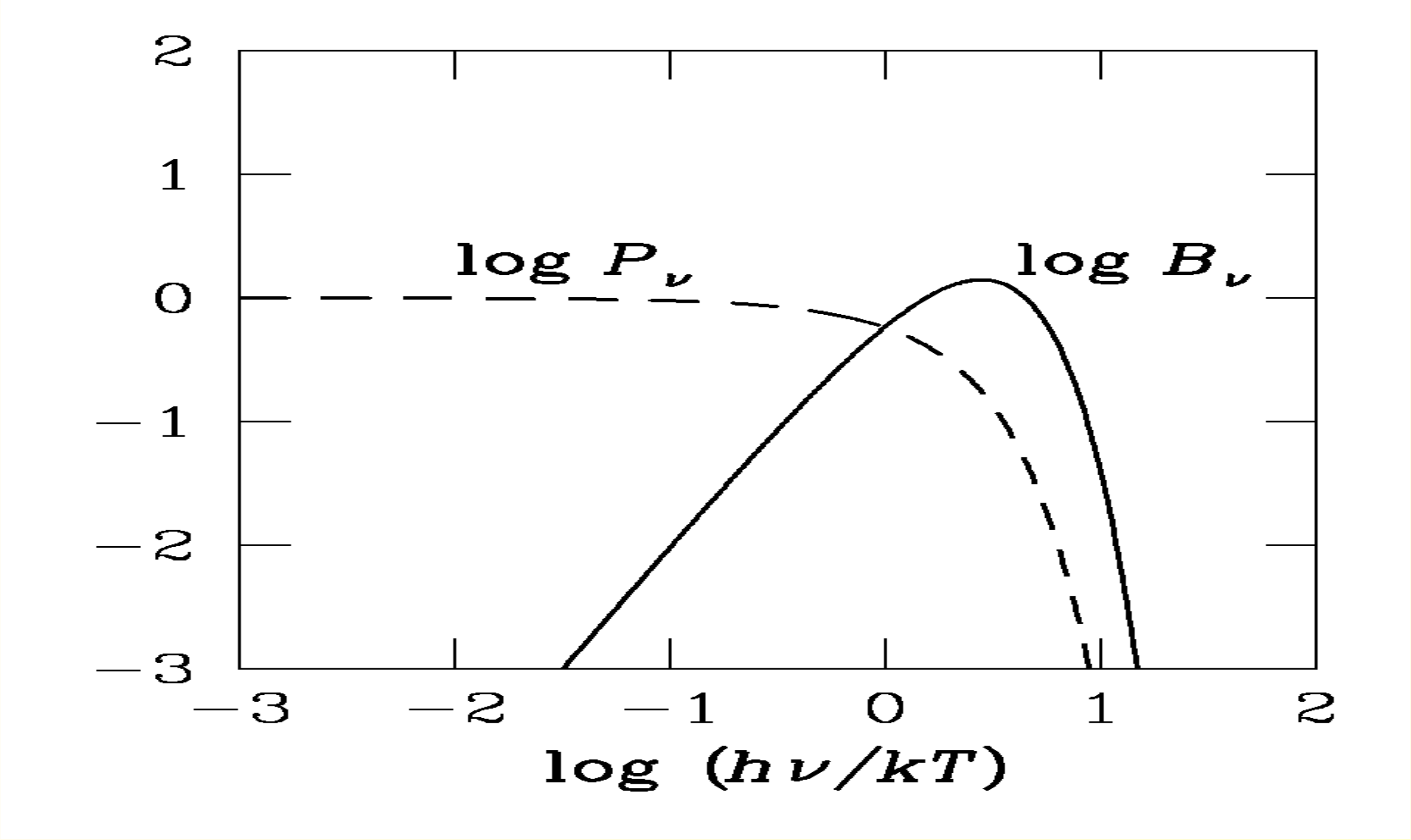
$$\frac{\partial B}{\partial \lambda}=0 \quad when \quad \lambda=\frac{2.9\text{mm}}{T(K)}$$



Power from warm resistor

- The resistor is the electrical equivalent of a black body in 1 dimension instead of 3

$$P(\nu,T)=\frac{h\nu}{e^{h\nu/kT}-1}\simeq kT \text{ at low frequencies}$$



Johnson-Nyquist Resistor noise

- Independant of resistance!
- Related to CCD dark current noise
- Depends on Temperature only at low freq.
- Comes from the Fluctuation-Dissipation theorem of statistical mechanics