Lecture 7

- Answers
- Accelerated charges - Larmor radiation

Answers from last time

- Very Long Baseline Interferometry
  - suppose a source has a flux density of 0.2Jy at 2.4GHz and 0.5Jy at 8GHz and has a size of 2 milliarcseconds ...
- Suppose we have a patch of galaxy radiating at 400K at 151MHz ; how many Jy would we receiver in a radio telescope beam of 2º

answers...

Larmor radiation derivations

Longair: *High Energy Physics*

- http://en.wikipedia.org/wiki/Larmor_formula#Radiation_reaction
- http://fermi.la.asu.edu/PHY531/larmor/larmor.html
- http://webphysics.davidson.edu/Applets/Retard/Retard.html
- and an animation:

  http://webphysics.davidson.edu/Applets/Retard/Retard.html

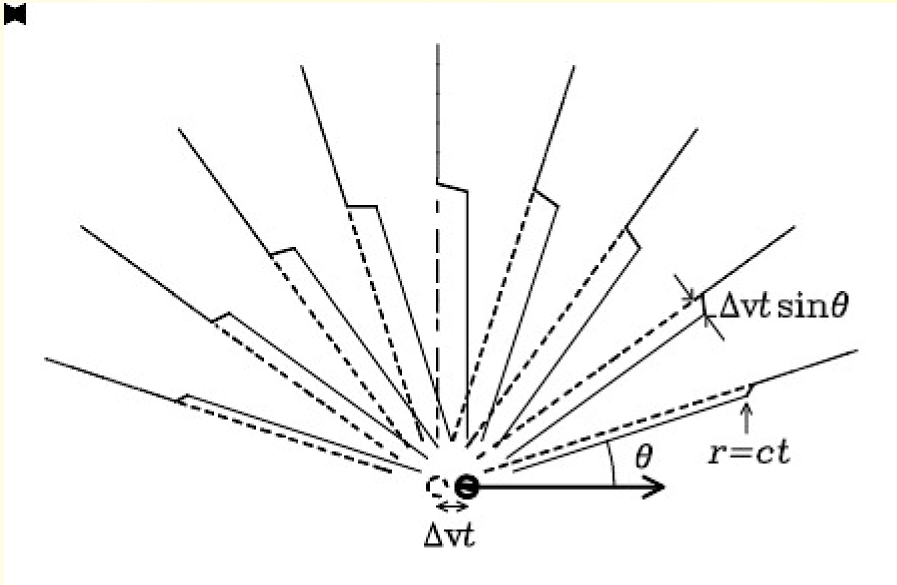
Radiation from accelerated charge

- from Maxwell's equations, consider an accelerated charge ($\Delta V/\Delta t$) and using retarded potentials
- we consider the low velocity case $V \ll c$ in a short time; if charge *q* is stationary we have purely radial field E_r, but in moving it we get a perpendicular component $E_\perp$
- if θ is the angle between acceleration vector and the direction of the observer

$$\frac{E_\perp}{E_r} = \frac{\Delta V\, t \sin(\theta)}{c\, \Delta t}$$

Larmor

- so if charge accelerates to you θ=0
  - no shift in field lines



Using Coulomb's law

- Coulomb field

$$E_r = \frac{q}{4\pi\epsilon r^2}$$

$$E\perp = \frac{q}{4\pi\epsilon r^2}\left(\frac{\Delta V}{\Delta t}\right)\frac{r\sin(\theta)}{c^2}$$

$$E\perp = \frac{q\dot{V}\sin(\theta)}{4\pi\epsilon r c^2}$$

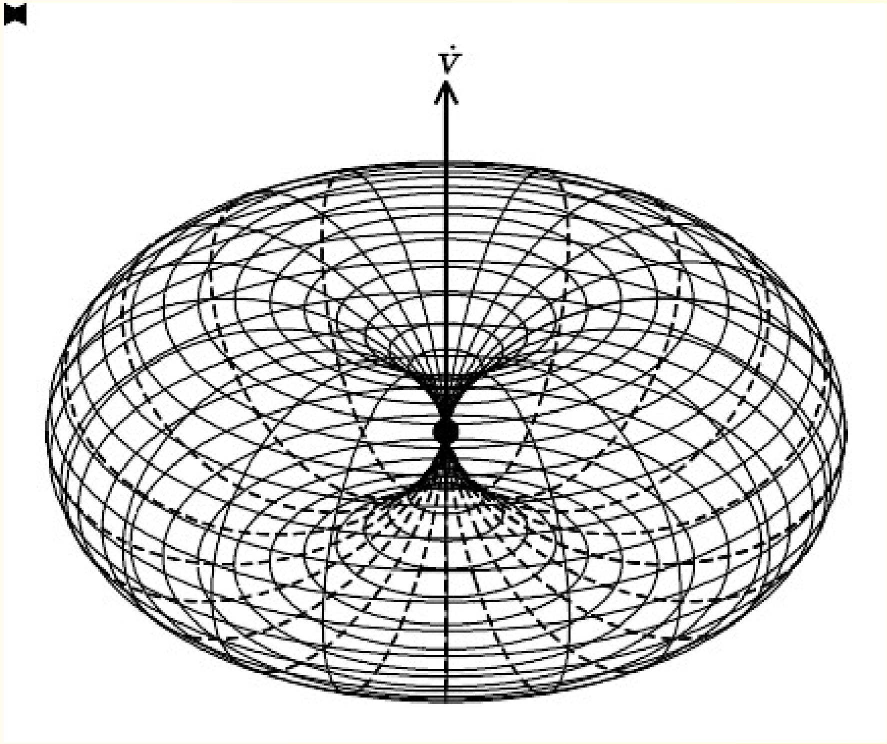
Poynting Flux

- Poynting flux $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu}$ $\mu \epsilon c^2 = 1$
- Equipartition of energy between electric and magnetic power

$$\epsilon E^2 = \frac{B^2}{\mu} \Rightarrow |B| = \frac{|E|}{c}$$

$$|\vec{S}| = \frac{E^2}{\mu c} = \epsilon c [\frac{q \dot{V} \sin(\theta)}{4 \pi \epsilon r c^2}]^2 = [\frac{q^2 \dot{V}^2}{16 \pi^2 \epsilon c^3}][\frac{\sin(\theta)^2}{r^2}]$$

How it looks (shown tilted by 60º)



- No radiation along acceleration axis
- maximum radiation transverse to axis
- rotational symmetry
- Polarized

Emission over the sphere

$$power\ P = \int_{sphere} |\vec{S}|$$

- taking just the geometrical part and cylindrical transform

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{pi} \frac{\sin^2(\theta)}{r^2} r\sin(\theta)\, d\theta\, r\, d\phi = 2\pi \int_0^{\pi} \sin^3(\theta) = 8\pi/3$$

- so Power emitted =

$$\frac{q^2 \dot{V}^2}{6\pi\epsilon c^3} \quad \text{in SI units}$$

$$\frac{2q^2 \dot{V}^2}{3c^3} \quad in\, CGS$$

Notes

- I have assumed vacuum (pretty accurate)
- The full relativistic version needs Liénard-Wiechert potentials

  http://fermi.la.asu.edu/PHY531/larmor/index.html
- http://www.jpoffline.com/physics_docs \

  /y3s6/electrodynamics_ln.pdf

  chapter 4
- Depends on charge and acceleration. For most objects the electrons accelerate most as they are lightest and so accelerate more for the same force, so they totally dominate emission.