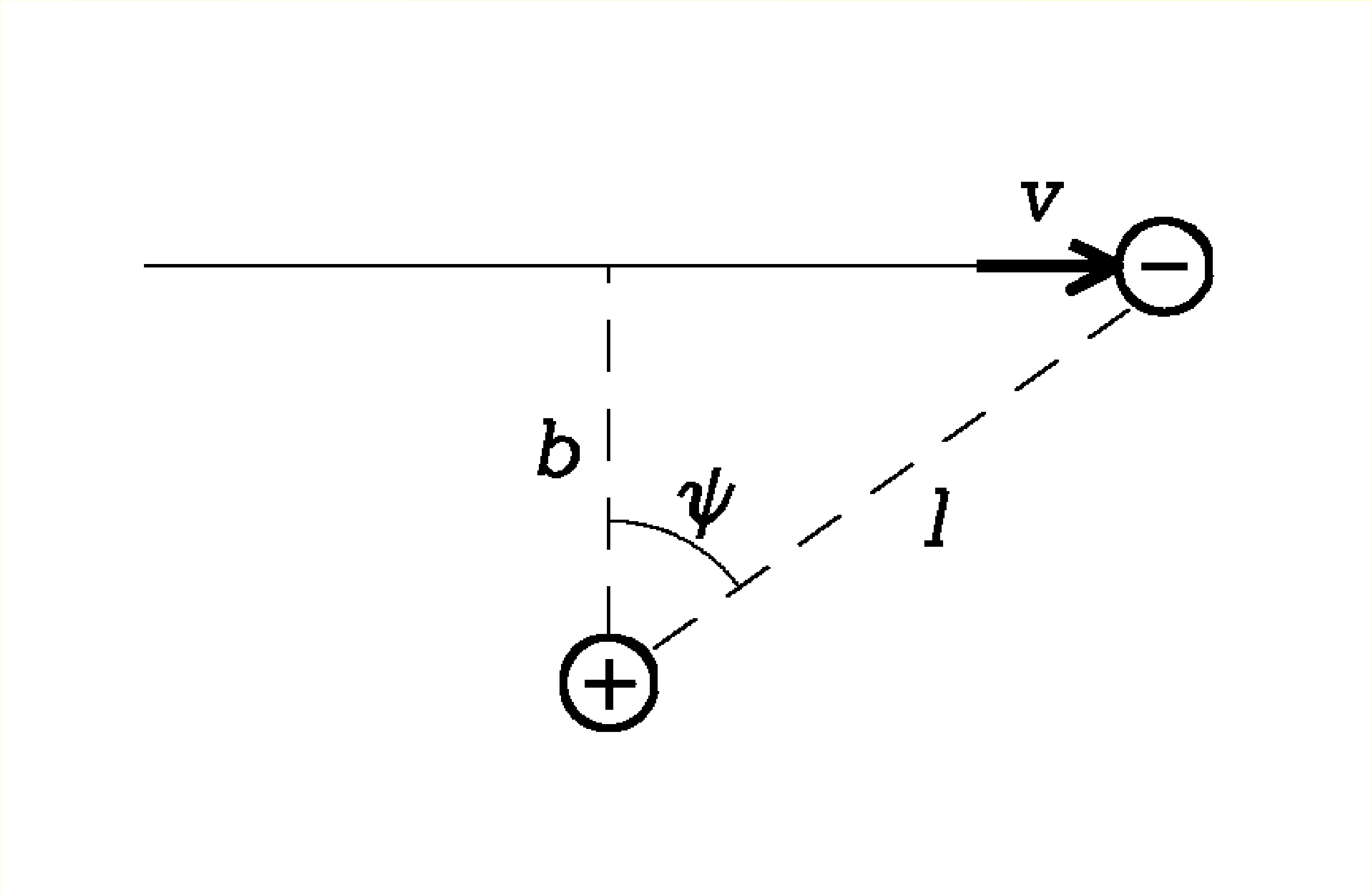
Lecture 8 Free-Free

- Also known as '*bremstrahlung*' – braking radiation
- Starting with our Larmor formula for power radiated by an accelerating charged particle

Consider an electron passing an ion



The velocity is changed

- Moving past the ion's electric field the electron's velocity changes (=acceleration)
- Collision time $\tau = b/v$ mean thermal energy of electron in a hot plasma = 3kT/2
  - for HII region at 10^4K this is about $2x10^{-19}$J ~1ev
  - much higher than energy of a radio photon (10GHz equivalent to $6.6x10^{-24}$J)
  - So radio photons are from *tiny* deflections

formulae

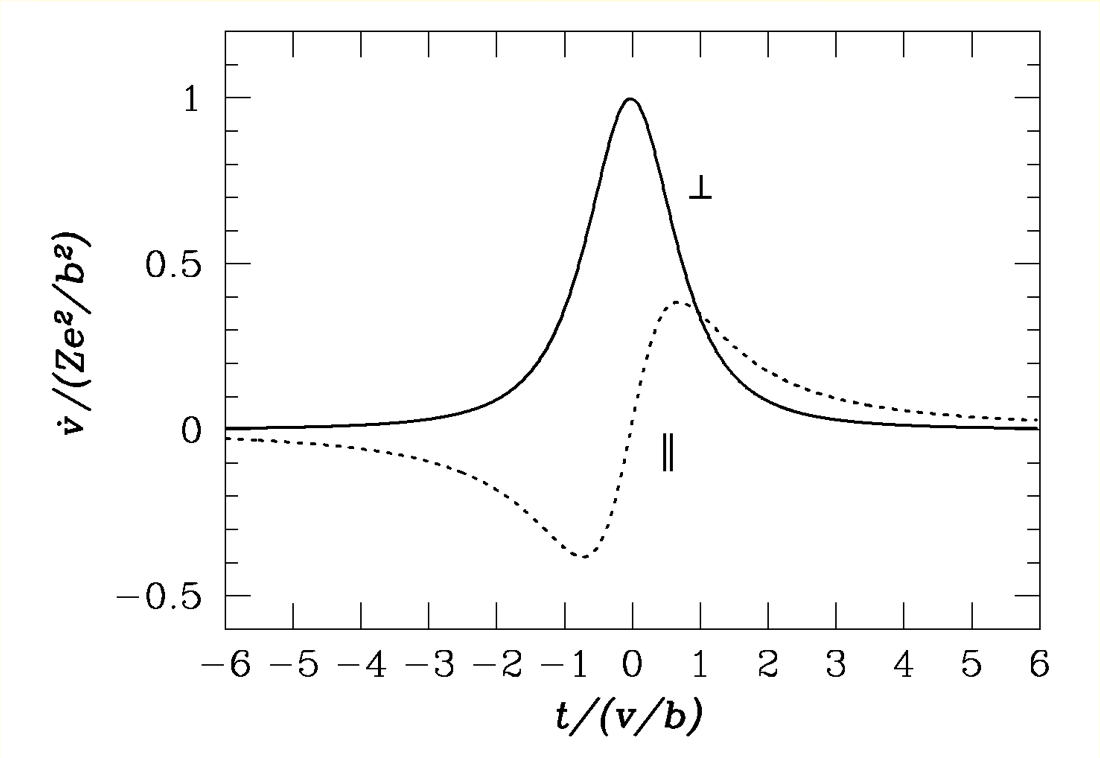
- for ion charge Zq

$$F_{\parallel} = m_e \dot{v}_{\parallel} = \frac{Zq^2}{l^2} \sin(\psi) = \frac{Zq^2}{b^2} \sin(\psi) \cos^2(\psi)$$

$$F_{\perp} = m_e \dot{v}_{\perp} = \frac{Ze^2}{l^2} \cos(\psi) = \frac{Ze^2}{b^2} \cos^3(\psi)$$

Change in velocity

- looks like



- perpendicular acceleration dominates (so when squared even more so)

Consider perpendicular acceleration

- instantaneous radiated power

$$P = \frac{q^2 \dot{v}_\perp^2}{6\pi \epsilon c^3} = \frac{Z^2 q^6}{6\pi \epsilon m_e} \left[\frac{\cos^3(\psi)}{b^2} \right]^2$$

- so over the total time of the interaction the total energy radiated

$$W = \int_{-\infty}^{\infty} P\, dt$$

- approximations: velocity hardly changes (roughly equal going towards ion and away from ion)

...

$$v = \frac{dx}{dt} \qquad \text{and} \qquad \tan\psi = \frac{x}{b}$$

$$v = \frac{b \quad d\psi}{\cos^2\psi \quad dt}$$

$$dt = \frac{b \; d\psi}{\cos^2(\psi)}$$

so emission becomes $\frac{q^6 Z^2}{6\pi\epsilon m_e} \int_{-\infty}^{\infty} \cos^6(\psi)\, dt$

integrating

changing variable $W = \frac{Z^2 q^6}{6\pi \epsilon c^3 m_e^2 b^4} \int_{-\pi/2}^{\pi/2} \frac{b \cos^6(\psi)}{v \cos^2(\psi)} d\psi$

$$W = \frac{Z^2 q^6}{3\pi \epsilon c^3 m_e^2 b^3 v} \int_0^{\pi/2} \cos^4(\psi) d\psi$$

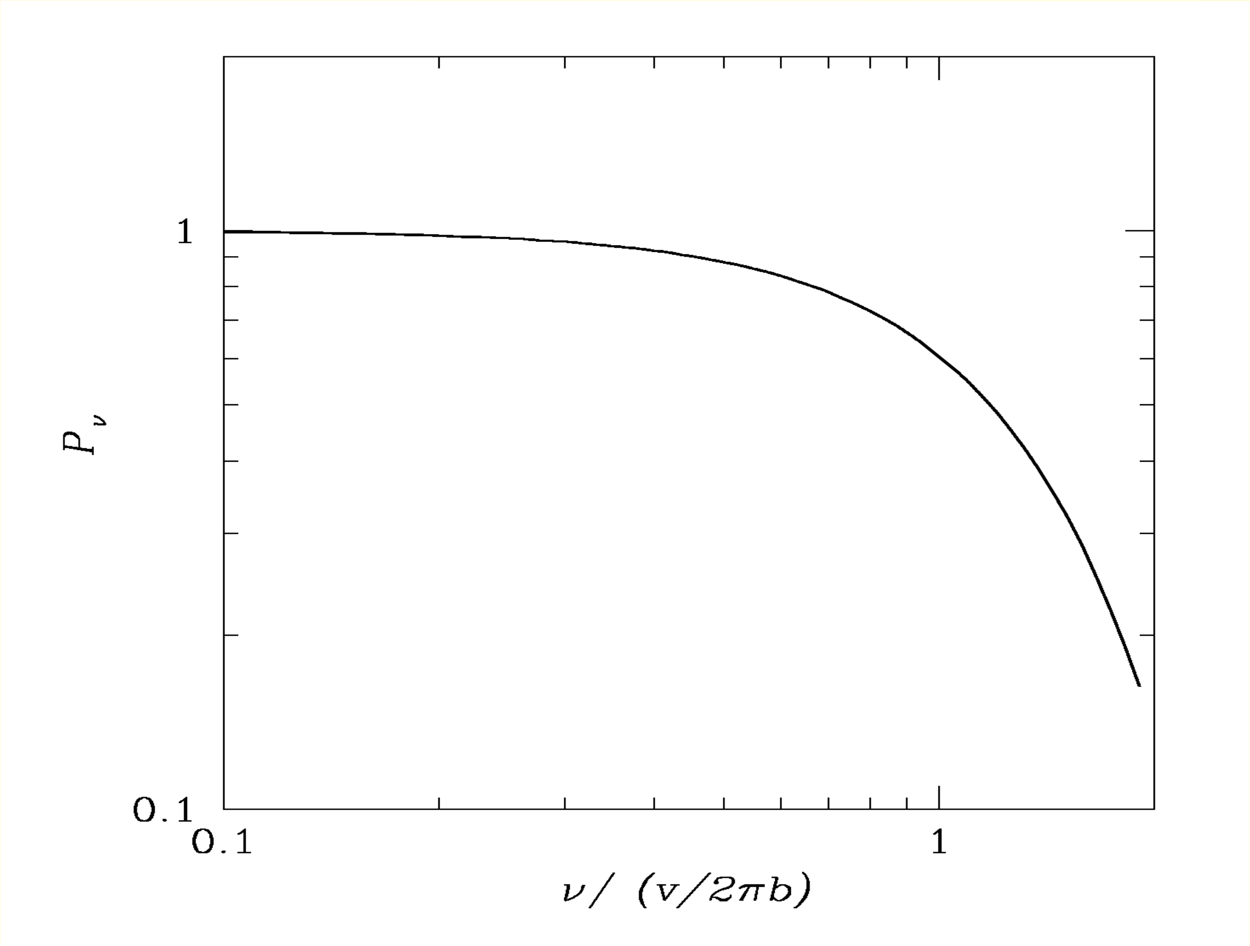
and integral

$$\int_0^{\pi/2} \cos^4(\psi) d\psi = \frac{3\pi}{16}$$

so $W = \frac{Z^2 q^6}{16 \epsilon c^3 m_e} [\frac{1}{b^3 v}]$

Short pulse = Wide spectrum

- pulse width τ => spectrum up to 1/2πτ



For HII region

- Typical values T ~ 10^4K (v ~ 700km/s) b ~ 1nm gives cutoff in infrared
- We can make an approximation for radio that up to a maximum frequency v/2πb

$$W_\nu \sim \frac{W}{\nu_{max}} = \frac{Z^2 q^6}{16\epsilon c^3 m_e^2 b^3 v}\left[\frac{2\pi b}{v}\right]$$

$$= \frac{\pi Z^2 q^6}{8\epsilon c^3 m_e^2}\left[\frac{1}{b^2 v^2}\right]$$

electrons passing

- Number of electrons passing in a shell *b* to *b*+*db* at velocity v to v+dv =

$$N_e(2\pi\, b\, db)\, v\, f(v)\, dv$$

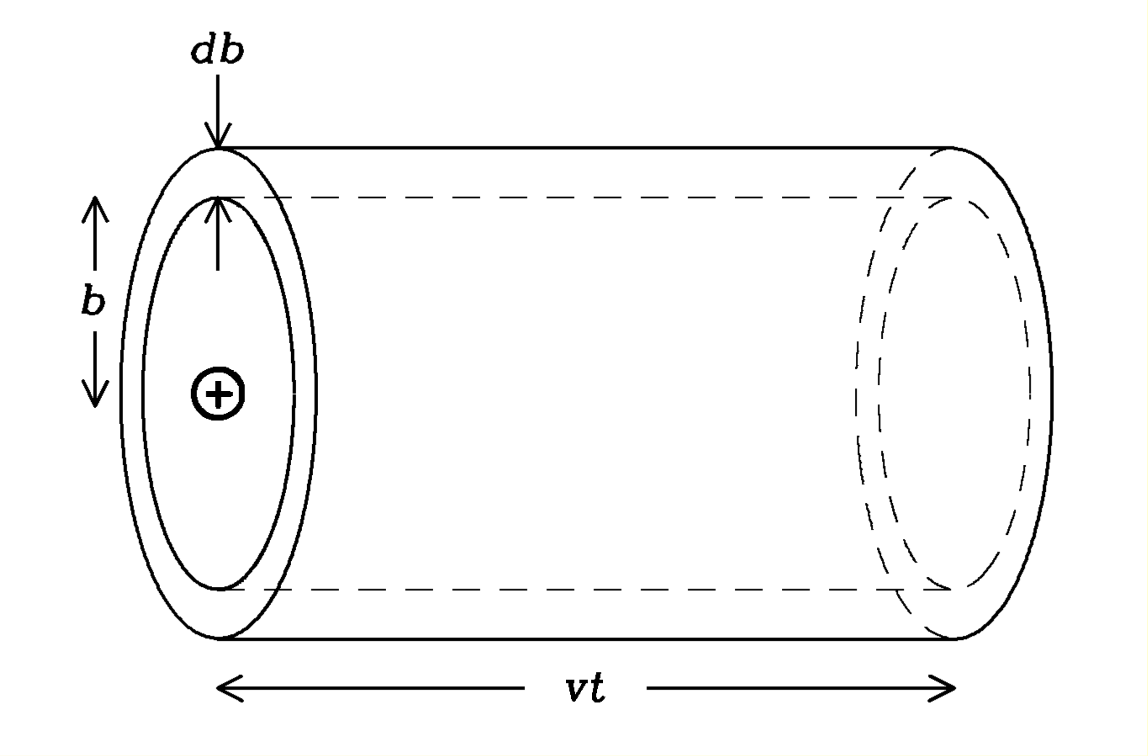
- Interactions per unit volume - multiply by ion density N_i to calculate emission per unit volume

- assume f(v) is Maxwellian

$$f(v)=\frac{4v^2}{\sqrt{(\pi)}}\left[\frac{m}{2\mathrm{kT}}\right]^{3/2}\exp\left(\frac{-mv^2}{2\mathrm{kT}}\right)$$

Radio from HII region

- Equipartition of energy suggests we can approximate to stationary ions during an interaction (they are heavier than electrons). Consider the cylindrical shell



So emissivity

- emissivity per unit volume per steradian after integrating

$$\varepsilon = \frac{\pi Z^2 q^6 N_e N_i}{16 \epsilon_0 c^3 m_e^2} \sqrt{\left(\frac{2m_e}{\pi kT}\right)} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

- Assuming that the plasma is hot enough (>>1000K) for us to ignore quantum mechanical effects

$$b_{min} \sim \frac{Zq^2}{4\pi\epsilon m_e v^2} \sim \frac{Zq^2}{3\pi\epsilon kT} \qquad b_{max} \sim \frac{v}{2\pi\nu}$$

- For typical HII regions $b_{max}/b_{min} \sim 10^4$ so the log factor is roughly 10 (9.2)
- so emissivity is proportional to 1/sqrt(T)

A slightly better approximation

- b_{max} varies very slowly with frequency so a better approximation is a variation with $\nu^{-0.1}$

Absorption

- Using the relationship $\kappa_\nu = \frac{\varepsilon_\nu}{B_\nu(T)}$
- for radio band $\kappa_\nu = \frac{\varepsilon_\nu c^2}{2kT\nu^2}$
- Using all the above approximations we get

$$\kappa_\nu = \frac{1}{\nu^2 T^{3/2}} \left[\frac{Z^2 q^6}{4\pi\epsilon c} N_e N_i \frac{1}{\sqrt{(2\pi(m_e k)^3)}} \right] \frac{\pi^2}{4} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

- In practice nobody uses that!
- However

$$\kappa_\nu \propto \nu^{-2.1}$$

$$\varepsilon_\nu \propto \nu^{-0.1}$$

Optical depth

$$\tau_\nu = \int -\kappa \, ds \propto \int \frac{N_e N_i}{\nu^{2.1} T^{3/2}} ds$$

Slightly more practical

$$\tau_\nu = 3.38 \times 10^{-7} \left(\frac{T}{10^4 K}\right)^{-1.35} \left(\frac{\nu}{GHz}\right)^{-2.1} \left(\frac{EM}{pc\,cm^{-6}}\right)$$

Change with optical depth

- If optically thin (high freq)

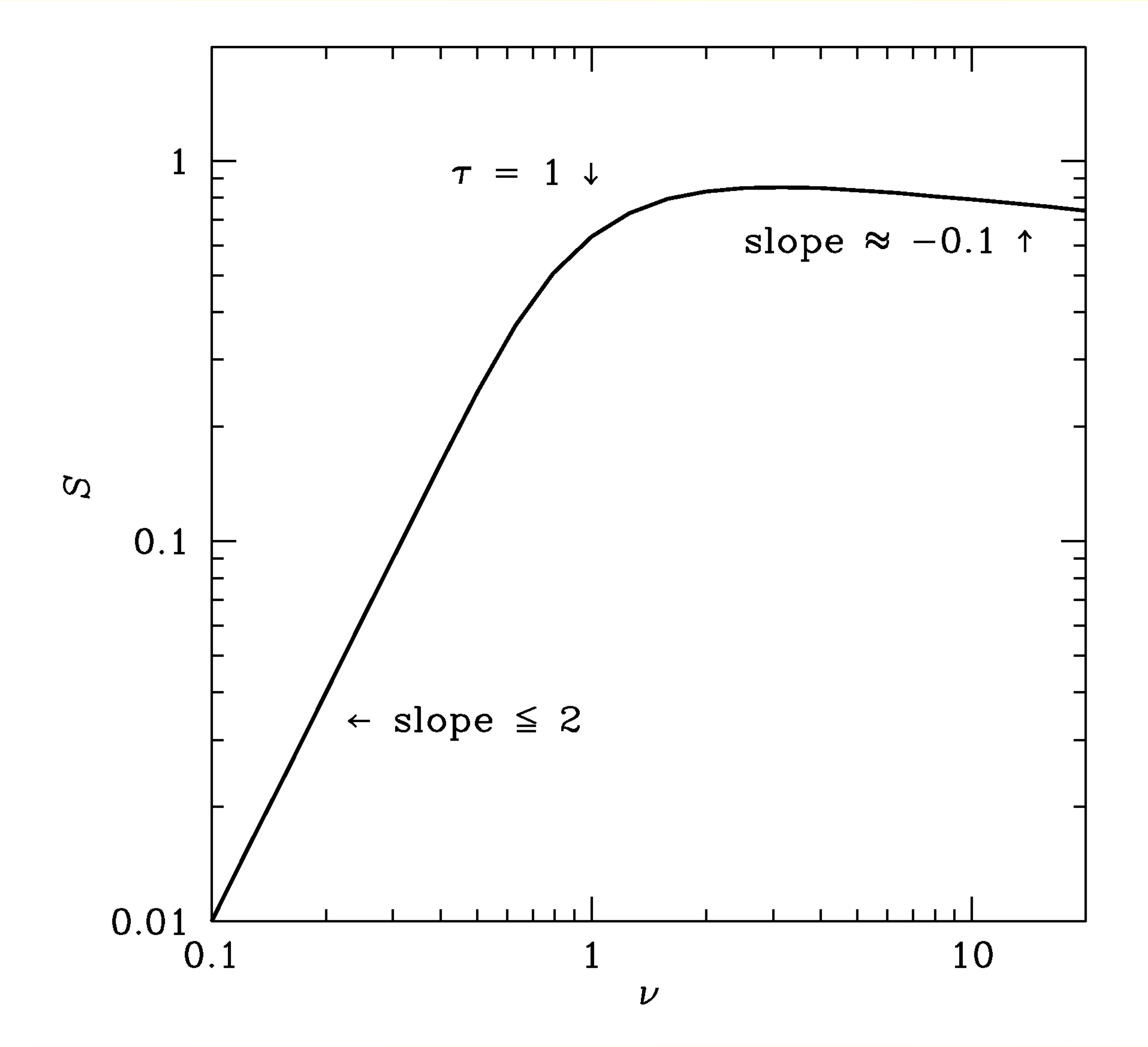
$$(1 - e^{-\tau}) \approx \tau$$

$$S_\nu \propto \frac{2kT\nu^2}{c^2}\tau_\nu \propto \nu^{-0.1}$$

- If optically thick (low frequency) looks like Blackbody

$$S_\nu \propto \frac{2kT\nu^2}{c^2} \propto \nu^2$$

Spectrum



spectrum

- Transparent at high frequency with gentle roll of of power
- Opaque (high optical depth) at low frequency with a spectrum quite like a black body
- Frequency of where optical depth =1 depends on density and temperature

In a horrible mix of units

Emission Measure $\frac{EM}{pc\ cm^{-6}} \equiv \int_{los} \left(\frac{N_e}{cm^{-3}}\right)^2 d\left(\frac{s}{pc}\right)$

optical depth $\tau_\nu = 3.01 \times 10^{-2} \left(\frac{T_e}{K}\right)^{-3/2} \left(\frac{\nu}{GHz}\right)^{-2} \left(\frac{EM}{pc\ cm^{-6}}\right) g_{ff}$

where g_{ff} is the Gaunt free-free factor

$$g_{ff} \sim \ln\left[4.955 \times 10^{-2} \left(\frac{\nu}{GHz}\right)^{-1}\right] + 1.5 \ln\left(\frac{T_e}{K}\right)$$

More practical formulae

- We can estimate the number of ionizing photons (N_{Ly}) per second from optical observations from the free-free luminosity at high frequency (optically thin)

$$\left(\frac{N_{Ly}}{s^{-1}}\right)=6.3\times 10^{52}\left(\frac{T_e}{10^4\,K}\right)^{-0.45}\left(\frac{\nu}{GHz}\right)^{0.1}\left(\frac{L_\nu}{10^{20}\,W\,Hz^{-1}}\right)$$