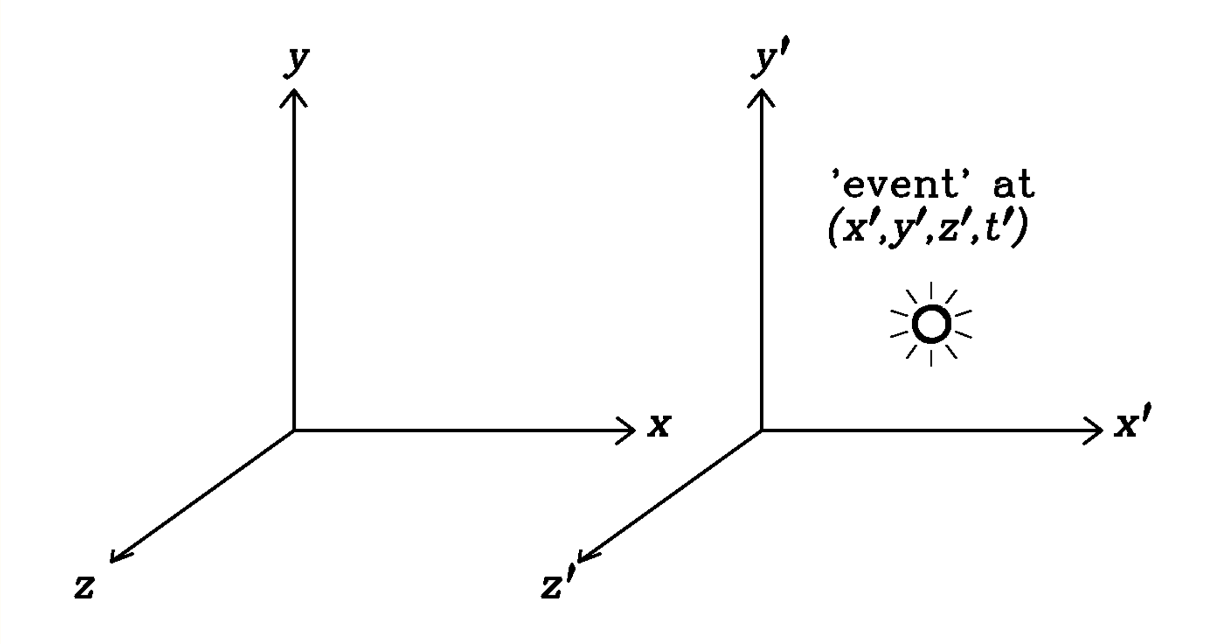
Synchrotron Radiation

- also 'magnetobremstrahlung'
- it is an intrinsically relativistic version of cyclotron, so you need to define frames
- **Much** more power is radiated, so this is more relevant to bright sources
- Good article in Wikipedia
- preferably treat using Liénard-Wiechert potentials

Special relativity



$$x=\gamma(x'+vt') \quad y=y' \quad z=z' \quad t=\gamma(t'+\beta x'/c)$$
$$x'=\gamma(x-vt) \quad t'=\gamma(t-\beta x/c)$$

$$\beta\equiv v/c \qquad \gamma\equiv\frac{1}{\sqrt{(1-\beta^2)}}$$

High energy cosmic rays

- We can observe high energy electrons enter the atmsophere with 10^9 to 10^{14}eV
- Rest energy from mass of electron ($9x10^{-31}$kg) = $8.2x10^{-14}$J = 0.51MeV
- so γ from 2000-200000000 - this is ultrarelativisitic

Compare with cyclotron

- Expect orbital speed to be reduced with factor γ (mass increases)
- Radius of orbit increases with factor γ
- In the field of our galaxy (0.5nT) this would be roughly 2hrs orbit with radius 2AU for $\gamma=10^5$
- That would not radiate much...

BUT

- The power radiated is increased by γ^2 and the sinusoid we had for cyclotron is turned into narrow spikes in time (broad band in frequency)
- We want to use the Larmor formula

$$P = \frac{q^2 a^2}{6\pi \epsilon c^3}$$

- But from out frame the velocity and acceleration changes
- Apparent speed up by γ and acceleration by γ^2

So power...

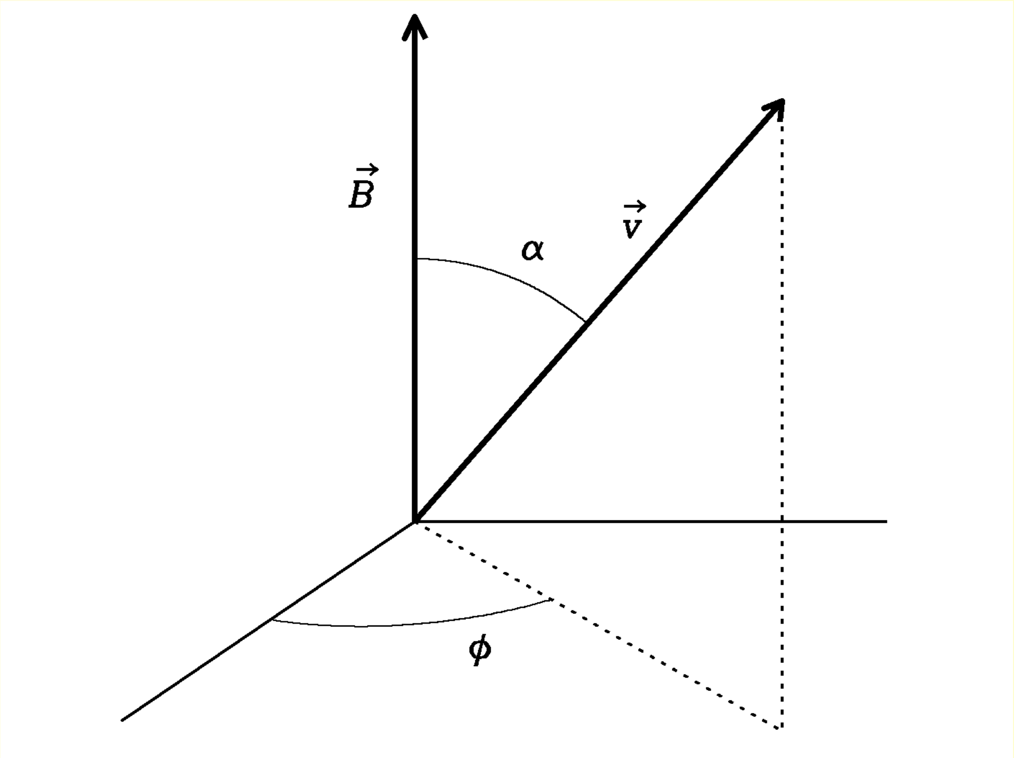
- Power loss is the same in all frames (if we see more energetic photons their arrival rate decreases)
- However our 'heavier' electron moves more slowly around the magnetic field lines

Power

$$P = \frac{q^2 a_\perp^2 \gamma^4}{6\pi\epsilon c^3}$$

but $\quad \omega_B = \frac{eB}{\gamma m} \quad$ and $\quad a_\perp = \omega_B V_\perp$

so $\quad P = \frac{q^4 B^2 \gamma^2 V^2 \sin^2(\alpha)}{6 \quad \pi\epsilon c^3}$



Simplification

- Lump together the electron constants (mass, charge etc) into Thomson cross section σ_T
- Lump B^2 term into magnetic field energy density

$$U_B = \frac{B^2}{2\mu}$$

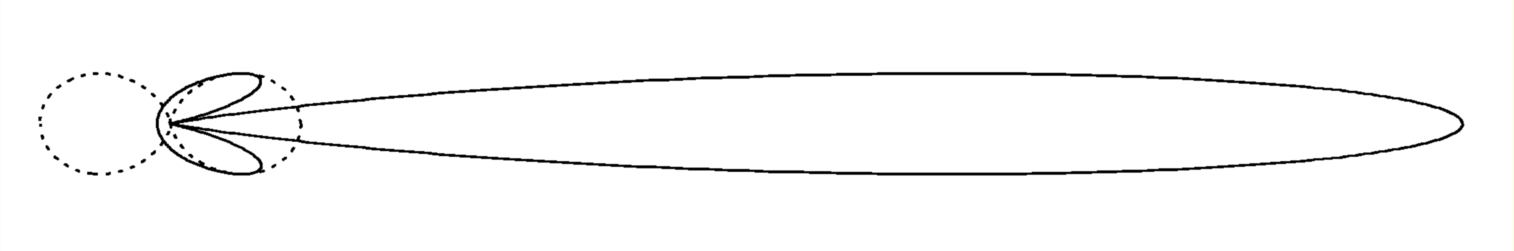
$$P = 2\sigma_T \beta^2 \gamma^2 c U_B \sin^2(\alpha)$$

- For random pitch angles in 3D space $\sin^2(\alpha) = 2/3$ so power emitted per electron is

$$P = \frac{4}{3}\sigma_T \beta^2 \gamma^2 c U_B$$

Direction

- What was a orginally a torus for Larmor radiaton is now also affected by relativity we get a beaming effect

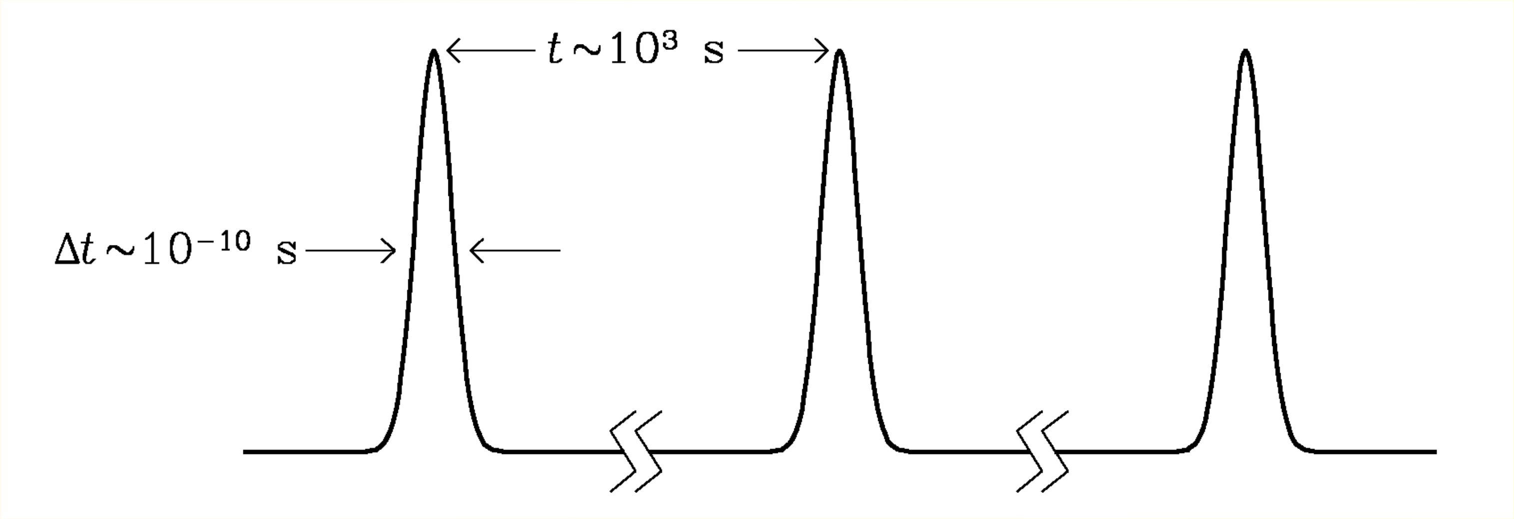


- Nulls that were at top and bottow now appear at

$$\sin(\theta) = \frac{1}{\gamma} \quad \text{which for large gamma gives} \quad \theta \sim \frac{1}{\gamma}$$

Pulsing

- The bulk of the power we observed is only from the tiny fraction of the time that the electron is coming towards us! $\Delta t \sim \frac{1}{\gamma^2 \omega_G}$
- So for our example of an electron with $\gamma = 10000$ in the galactic field we would see



Narrow pulse = wide spectrum

- For our example this means that there will be radiation up to 1GHz in frequencies spaced by 1mHz
- In practice the power emitted at low frequencies is fairly flat and tapers off at high frequencies

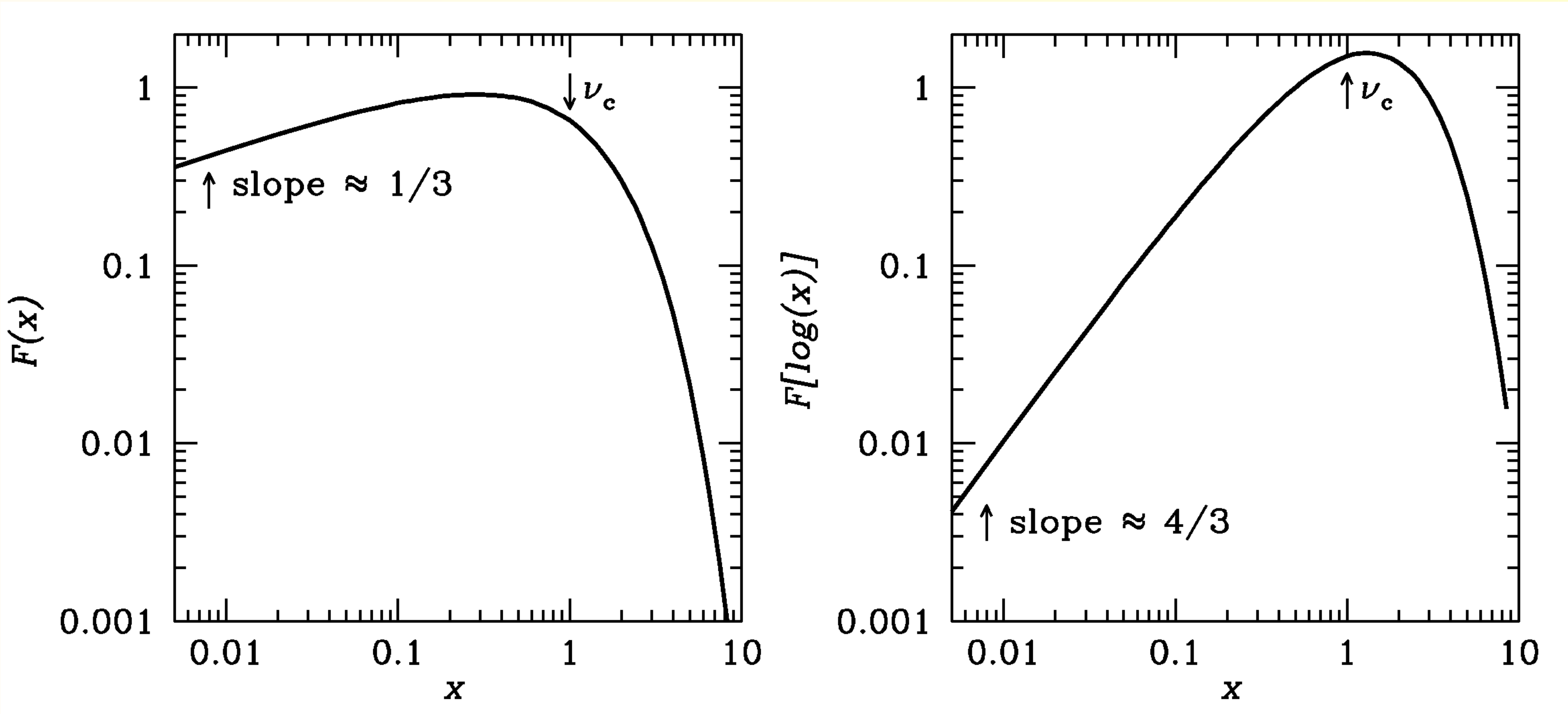
$$\nu_{max} \sim \gamma^2 \frac{eB}{m}$$

Spectrum

- See Pacholczyk: "*Radio Astrophysics*" or Rybicki & Lightman: "*Radiative Processes in Astrophysics*" or even Ginzburg and Syrovatskii
- Below a critical frequency power rises and above it the power drops

$$\nu_c = \frac{3}{2} \frac{\gamma^2 eB \sin(\alpha)}{m}$$

Calculated spectra



Approximation

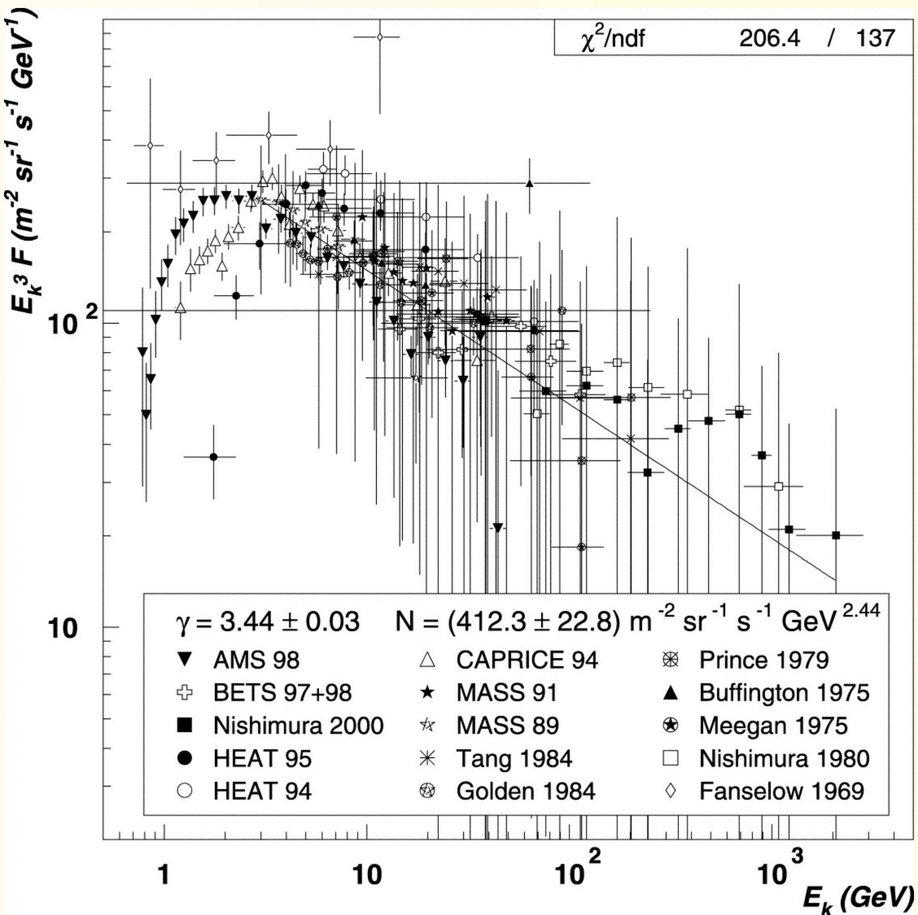
- logarithmic slope for single electron γ

$$\frac{d(\log P)}{d(\log \nu)} \quad 1/3$$

- most power is emitted near the peak

Realistic distribution of γ

- For our galaxy there is an approximate power law distribution of energies above a low-energy cutoff $N(E)\,dE \sim K\,E^{-\delta}\,dE$ with $\delta \sim 2.4$



More crude approximations...

Emitted power $$P = \frac{4}{3}\sigma_T \beta^2 \gamma^2 c U_B$$

mostly emitted at $$\nu = \gamma^2 \nu_G$$

so using $$\varepsilon_\nu d\nu = -\frac{dE}{dt} N(E)\, dE$$

and some manipulation ...

$$\varepsilon \propto B^{(\delta+1)/2} \nu^{(1-\delta)/2}$$

We define a spectral index α

- Flux density proportional to $\nu^{-\alpha}$
- NOT the same as pitch angle α

$$\alpha = \frac{\delta - 1}{2} \quad \text{and for our galaxy that gives} \quad \alpha = 0.7$$

$$\varepsilon \propto B^{1.7} \nu^{-0.7}$$

- This value of α about 0.7 is typical of many extragalactic synchrotron sources (probably related to the shock acceleration mechanism)

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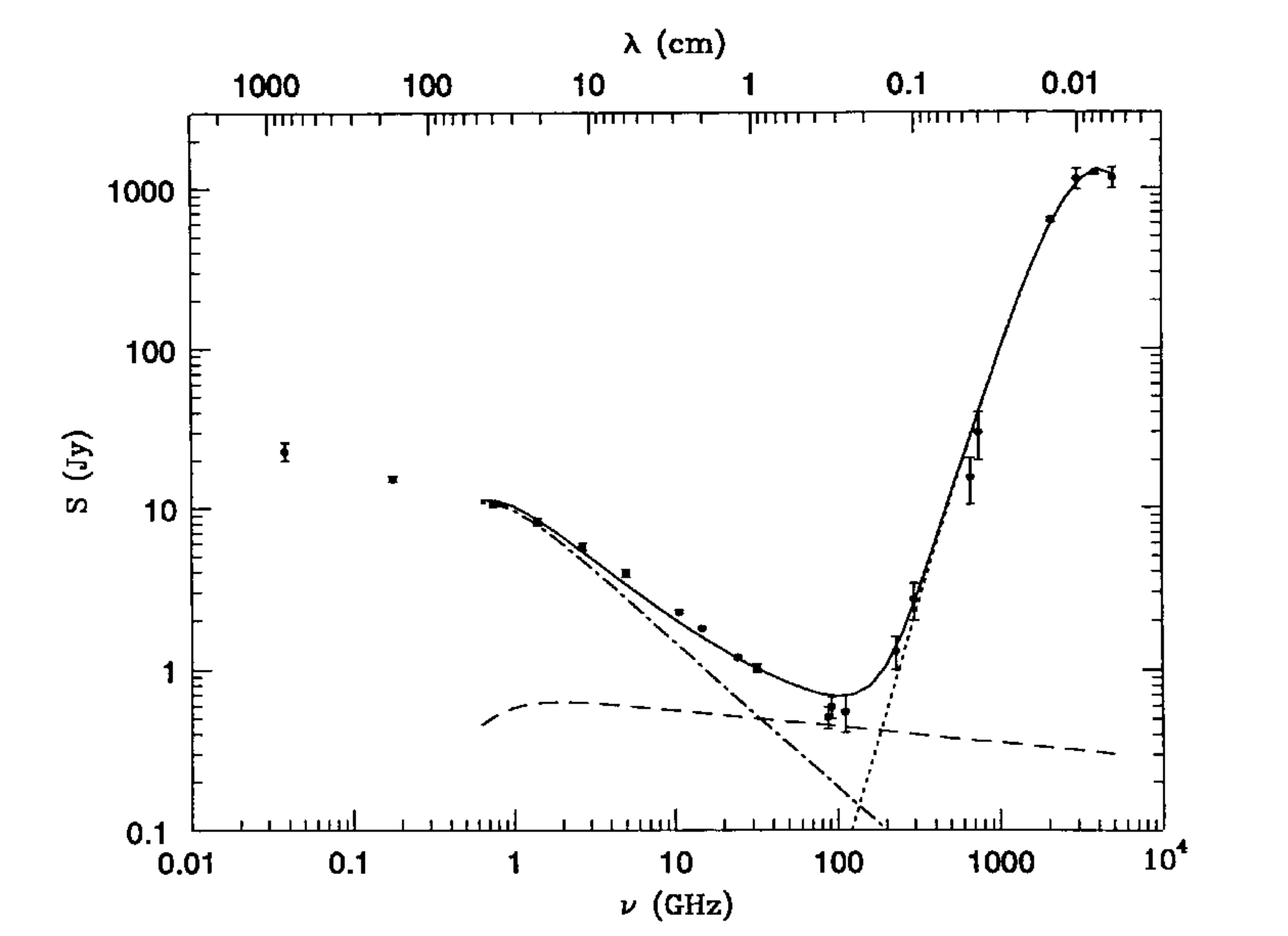


λ (cm)
1000
100
10
1
0.1
0.01
1000
100
10
1
0.1
S (Jy)
0.01
0.1
1
10
100
1000
10^4
ν (GHz)

More synchrotron...

- How much energy is involved
- see also http://asd.gsfc.nasa.gov/Volker.Beckmann/school/download/Longair_Radiation2.pdf

Energy in particles vs Luminosity

In electrons $U_e = \int_{E_{min}}^{E_{max}} E N(E)\, dE$

Luminosity $L = \int_{\nu_{min}}^{\nu_{max}} L(\nu)\, d\nu$

subsitituting $N(E) = KE^{-\delta}$ and using $-dE/dt \propto B^2 E^2$
as well as for a given frequency $E \propto 1/\sqrt{(B)}$

$$\frac{U_e}{L} \propto B^{-3/2}$$

So if we observe a luminosity

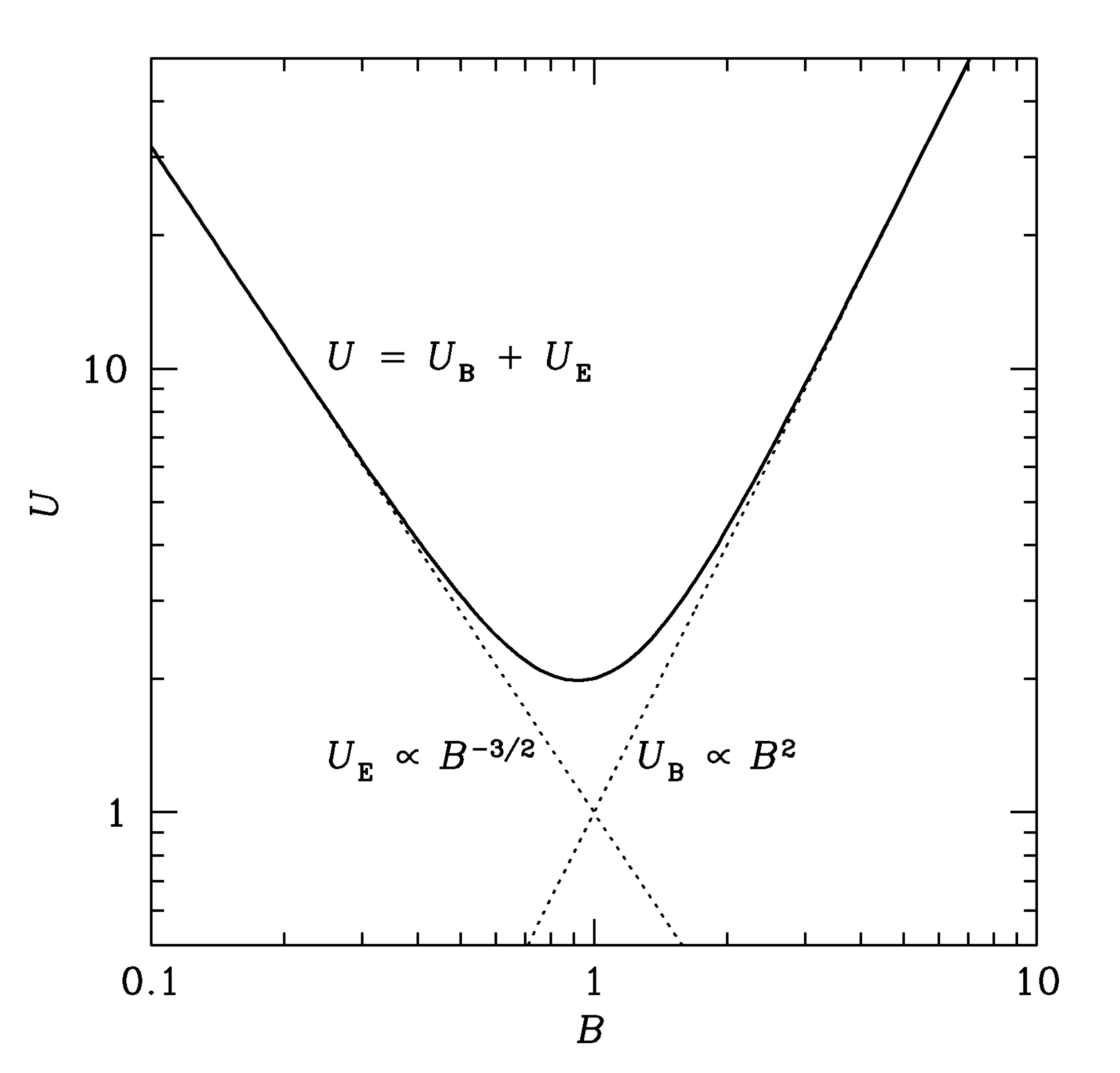
$$U_e \propto B^{-3/2}$$

and $$U_B \propto B^2$$

Assume other particles

we assume other particles with the same power law so

$U = U_e(1+\eta) + U_B$



so called "equipartition" or "minimum enenergy"

- Actually a minimum when

$$\frac{\textit{particle energy}}{\textit{magnetic field energy}}=\frac{U_e(1+\eta)}{U_B}=\frac{4}{3}$$

- Equipartition is (where the would be equal) is plausible on physical grounds
- If we are far off the energy demands get serious
- but we can only ~~guess~~ estimate η
- and we have to estimate cutoffs for the power law

In astronomer units...

- If the volume of the source is V (m^3) and luminosity L(ν) at ν Hz for typical spectrum and assumption of cutoffs

$$Energy = 3\times10^{6}(1+\eta)^{4/7} V^{3/7} \nu^{2/7} L_{\nu}^{4/7} \quad J$$

$$B_{min} = 1.8\left[\frac{(1+\eta)L_{\nu}}{V}\right]^{2/7} \nu^{1/7} \quad Tesla$$

Another version

- Energy density:
  - ***S*** in Jy
  - ***Ω*** in steradians
  - ***l*** is length through the source (kpc)

$$U = 1.29 \times 10^{9} \left[\frac{S}{\Omega\, l} \left(\frac{\nu}{10\mathrm{MHz}} \right)^{\alpha} \frac{1 - 10^{1.5 - 3\alpha}}{2\alpha - 1} \right]^{4/7} \quad J/m^{3}$$

$$B = 3.72 \times 10^{4} \left[\frac{S}{\Omega\, l} \left(\frac{\nu}{10\mathrm{MHz}} \right)^{\alpha} \frac{1 - 10^{1.5 - 3\alpha}}{2\alpha - 1} \right]^{2/7} \quad Tesla$$