Synchrotron Self-Compton: Diagnostic

If we are confident that the spectrum of a particular source is produced by the SSC mechanism, we can estimate a few parameters. We showed earlier that observations of the self-absorbed part of the synchrotron spectrum can yield a value for the magnetic field. Note that the observed flux for a source with known angular size is (self-absorbed part):

$$\boxed{F_s(\nu) \propto \Theta_s^2 \frac{\nu^{5/2}}{B^{1/2}} \quad (\text{watt m}^{-2}\ \text{Hz}^{-1})}$$

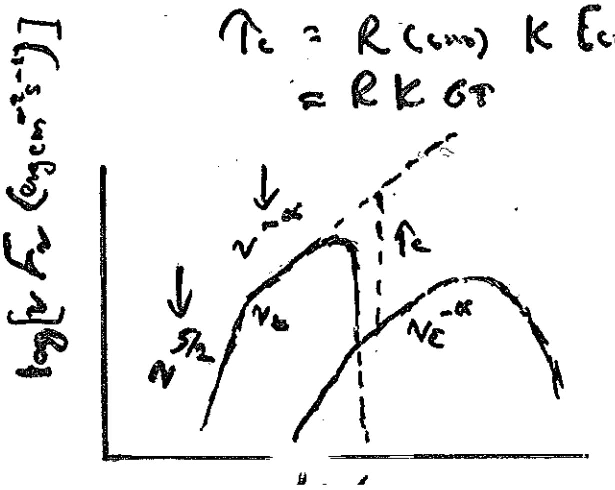
$$\Rightarrow \quad B \propto \frac{\Theta_s^4 \nu^5}{F_s^2(\nu)}$$

Observations of the optically thin part of the synchrotron spectrum reveal a flux

$$F_s(\nu) = \Theta_s^2 R K B^{1+\alpha} \nu^{-\alpha}$$

This allows us to determine:

$$\tau_e = R\,(\text{cm})\ K\,[\text{cm}^{-3} \text{ between } (\gamma, \gamma+d\gamma)]\ \sigma_T\,(\text{cm}^2)$$
$$= R K \sigma_T$$



Just notice that on a νF_ν plot the different parts (↓) of the synchrotron spectrum will have different slopes as a result of ($\nu \times F_\nu$).

There is an alternative scenario to estimate physical parameters. Observe the source at the turn-over frequency. Then one observe the source at both the optically thick and thin part simultaneously. Then the flux at the turn-over frequency ν_T is just F_T. Then

$$B^{1/2} \propto \theta_s^2 \frac{\nu_T^{5/2}}{F_T}$$

$$B \propto \theta_s^4 \frac{\nu_T^5}{F_T^2}$$

Then also from

$$F_T \propto \theta_s^2 R K B^{1+\alpha} \nu_T^{-\alpha} \qquad \tau_c = R K \sigma_T$$

$$\propto \theta_s^2 \left(\frac{\tau_c}{\sigma_T}\right) B^{1+\alpha} \nu_T^{-\alpha}$$

$$\tau_c \propto \frac{F_T \sigma_T}{\theta_s^2 B^{1+\alpha} \nu_T^{-\alpha}}$$

$$\propto \frac{\sigma_T F_T \nu_T^{\alpha}}{\theta_s^2 B^{1+\alpha}}$$

Then one can use this estimate for τ_c to determine or constrain source parameters related to the SSC component of the emission.

Since

$$\overset{\nu_c=\nu_{ZC}}{\downarrow}\qquad\qquad\qquad\overset{\nu_c=\nu_{ZC}}{\downarrow}$$

$$\mathcal{E}_{ssc}(\nu_c) \propto \tau_c \; \mathcal{E}_{syn}(\nu_c) \qquad \left[F_\nu \propto \mathcal{E}_\nu \times R \times \Theta_s^2\right]$$

$$\Rightarrow F_{ssc}(\nu_c) \propto \tau_c \; F_{syn}(\nu_c) \qquad \uparrow \mathcal{E}_\nu \propto K B^{1+\alpha} \nu^{-\alpha}$$

$$\propto \tau_c \times \Theta_s^2 R K \, B^{1+\alpha} \nu_c^{-\alpha}$$

$$\propto \tau_c \times \Theta_s^2 \left(\frac{\tau_c}{\sigma_T}\right) B^{1+\alpha} \nu_c^{-\alpha}$$

$$\propto \Theta_s^2 \times \frac{\tau_c^2}{\sigma_T} B^{1+\alpha} \nu_c^{-\alpha}$$

$$\propto \Theta_s^2 \sigma_T^{-1} \left[\frac{F_T \nu_T^{\alpha}}{\Theta_s^2 B^{1+\alpha}}\right]^2 B^{1+\alpha} \nu_c^{-\alpha}$$

$$\propto \Theta_s^{-2} \sigma_T^{-1} \frac{F_T^2 \nu_T^{2\alpha}}{(B^{1+\alpha})^2} B^{1+\alpha} \nu_c^{-\alpha}$$

$$\propto \Theta_s^{-2} \sigma_T^{-1} F_T^2 \nu_T^{2\alpha} \left[\left[\frac{\Theta_s^4 \nu_T^5}{F_T^2}\right]^{(1+\alpha)}\right]^{-1} \times \nu_c^{-\alpha}$$

$$\propto \Theta_s^{-2} \sigma_T^{-1} F_T^2 \nu_T^{2\alpha} \left(\frac{\Theta_s^4 \nu_T^5}{F_T^2}\right)^{-(1+\alpha)} \nu_c^{-\alpha}$$

$$\propto \left[\Theta_s^{-2} \Theta_s^{-4(1+\alpha)}\right]\left[F_T^2 F_T^{2(1+\alpha)}\right]\left[\nu_T^{-5(1+\alpha)} \nu_T^{2\alpha}\right] \times \nu_c^{-\alpha}$$

$$\propto \left[\Theta_s^{-2-4-4\alpha}\right]\left[F_T^{2+2+2\alpha}\right]\left[\nu_T^{-5-5\alpha+2\alpha}\right] \nu_c^{-\alpha}$$

$$\propto \Theta_s^{-4\alpha-6} F_T^{2\alpha+4} \nu_T^{-3\alpha-5} \nu_c^{-\alpha}$$

$$\propto \Theta_s^{-2(3+2\alpha)} F_T^{2(\alpha+2)} \nu_T^{-(5+3\alpha)} \nu_c^{-\alpha}$$

↑ The more compact the source the higher the flux

The fact that a more compact source yields a higher flux for SSC process can be understood within the context that for a compact source the relativistic electrons and synchrotron photons have higher probability to interact.

There is however a problem with the method used. For the strongest radio-loud sources there is a significant difference between the predicted and observed flux by several orders of magnitude. This is because for the strong radio loud sources like blazars there is strong beaming which has not been taken into account in the analysis. This has to be incorporated for sources where there is relativistic motion (jets).

If the optically thick synchrotron flux

$$F^{syn}_{thick}(\nu) \propto \Theta_s^{\,2} \, \frac{\nu^{5/2}}{B^{1/2}}$$

has been Doppler boosted we see that since

$$B^{1/2} \propto \left(F^{syn}_{thick}(\nu)\right)^{-1}$$

$$\Rightarrow \quad B^{1/2} \to 0 \quad \text{for} \quad F^{syn}_{thick}(\nu) \to \infty$$

The boosted flux results in the B-field strength to be underestimated. Then if the B-field is underestimated we have to overestimate the particle density (relativistic) to explain the thin synchrotron emission. This can be seen from

The thin synchrotron flux and SSC flux

$$F^{syn}_{thin}(\nu) \propto \Theta_s^2 RK B^{1+\alpha} \nu^{-\alpha}$$

$$F_{ssc}(\nu) \propto \tau_c \quad F^{syn}_{thin}(\nu_c)$$

$$\tau_c = RK\sigma_T$$

From the eqn's above we see that for too low $\tau_c \propto (RK)$ B-field (underestimated) we have to boost to match the right synchrotron and SSC fluxes. We need to repeat the analysis keeping in mind that from strong radio sources like blazars Doppler boosting has to be incorporated.

The observed flux from boosted emission is

$$\boxed{F_\nu(obs) = \delta_D^3 \; F_\nu'(iso)}$$

Exercise:

Prove the identity above (see notes on Chapt 3!)

Then from the optically thick and thin fluxes

$$F'^{syn}_{thick} \propto \Theta_s^2 \frac{\nu'^{5/2}}{B'^{1/2}} \qquad \left[RK = \frac{\tau_c}{\sigma_T}\right]_{k'}$$

$$F'^{syn}_{thin} \propto \Theta_s^2 (RK)' B'^{(1+\alpha)} \nu'^{-\alpha}$$

in k' (in rest frame), we can get the boosted observed flux.

Then the boosted observed flux is

$$F_{thick}^{syn}(\nu) = \delta_D^3 \, F'^{syn}_{thick}(\nu')$$

$$= \delta_D^3 \left[\frac{\theta_s^2 \, \nu'^{5/2}}{B'^{1/2}} \right]$$

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$$= \delta_D^3 \, \frac{\theta_s^2}{B'^{1/2}} \left(\frac{\nu}{\delta}\right)^{5/2} \qquad \nu' = \frac{\nu}{\delta_D}$$

$$= \delta_D^{1/2} \, \frac{\theta_s^2}{B'^{1/2}} \, \nu^{5/2}$$

↑ in source

[The primed quantities are measured in K', the rest frame of the source]

For the optically thin part we need to adjust the eqn. for flux to take into account a power-law brightness of the source

$$F_{thin}^{syn}(\nu) = \delta_D^3 \, F'^{syn}_{thin}(\nu')$$

$$= \delta_D^3 \, F_0'^{syn} \, \nu'^{-\alpha}$$

$$= \delta_D^3 \, F_0'^{syn} \left(\frac{\nu}{\delta}\right)^{-\alpha}$$

$$= \delta_D^{3+\alpha} \, F_0'^{syn} \, \nu^{-\alpha}$$

$$= \delta_D^{3+\alpha} \, \underbrace{F'^{syn}_{thin}(\nu)}_{F_0'^{syn}\,\nu^{-\alpha}}$$

Then

$$F^{syn}_{thin}(\nu) = \delta_D^{3+\alpha} F'^{\,syn}_{thin}(\nu')$$

$$= \delta_D^{3+\alpha} \underbrace{\left[\theta_s^2 (RK) B^{1+\alpha} \nu^{-\alpha}\right]_{K'}}$$

This is the flux a co-moving observer will measure in the frame of the source at frequency ν.

At the turnover frequency we can estimate B:

Boosted flux

$$F_t = \delta_D^{1/2} \frac{\theta_s^2 \nu_t^{5/2}}{B'^{1/2}}$$

{Primed quantities are relevant for rest frame of source, i.e. K'}

$$B'^{1/2} = \delta_D^{1/2} \frac{\theta_s^2 \nu_t^{5/2}}{F_t}$$

$$B' = \delta_D \frac{\theta_s^4 \nu_t^5}{F_t^2}$$

[in K']

$$\Rightarrow B_{corr} = \delta_D B_{un\text{-}corr}$$

$$B_{un\text{-}corr} = \frac{\theta_s^4 \nu_t^5}{F_t^2}$$

{The incorrect B-field that is inferred from Doppler boosted flux.}

need to correct as well.

Now

$$F^{syn}_{thin}(\nu) \propto \delta_D^{3+\alpha} \theta_s^2 (RK)'_{corr} B_{corr}^{1+\alpha} \nu^{-\alpha}$$

$$\boxed{(RK)' = \frac{\tau_c'}{\sigma_T} = \frac{\tau_c^{corr}}{\sigma_T}}$$

Although $(RK) = (RK)'$ it has been given the incorrect value since our estimate of B was incorrect, see next page

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Now that we have corrected the magnetic field we need to correct for τ_c as well since

$$\tau_c \propto \frac{\sigma_T F_T \nu_T^{\alpha}}{\theta_s^2 B^{1+\alpha}}$$ [for parameters in frame of source] i.e K'

Since the measured flux was Doppler boosted it led to an underestimation of the magnetic field in the source. To correct τ_c we must incorporate correction of B.

$$\tau_c^{corr} = (\tau_c)_{K'}$$

$$\tau_c^{corr} = \frac{\sigma_T}{\theta_s^2} \frac{F_t' \nu_t'^{\alpha}}{B_{corr}^{1+\alpha}}$$

$$= \frac{\sigma_T}{\theta_s^2} \left(\frac{F_t}{\delta_D^3}\right)\left(\frac{\nu_t}{\delta_D}\right)^{\alpha} \Big/ B_{corr}^{1+\alpha}$$

$$= \frac{\sigma_T}{\theta_s^2} \delta_D^{-3} \delta_D^{-\alpha} \left(\frac{F_t \nu_t}{B_{corr}^{1+\alpha}}\right)$$

$$= \delta_D^{-3-\alpha} \frac{\sigma_T F_t \nu_t}{\theta_s^2 B_{corr}^{1+\alpha}}$$

$$\Rightarrow (RK)_{corr} = \frac{\tau_c^{corr}}{\sigma_T}$$

Therefore

$$F_{thin}^{syn}(\nu) \propto \delta_D^{3+\alpha} \theta_s^2 (RK)_{corr} B_{corr}^{1+\alpha} \nu^{-\alpha}$$

corrected to the frame of the source. They now reflect the correct values inherent to the source.

Now we can estimate the SSC flux:

The corresponding SSC flux

$$F_{ssc}(\nu_c) \propto \tau_c^{corr} \, F^{syn}_{thin}(\nu_c)$$

$$\propto \tau_c^{corr} \, \delta_D^{3+\alpha} \, \theta_s^{2} \, (RK)_{corr} \, B_{corr}^{1+\alpha} \, \nu_c^{-\alpha}$$

$$\propto \tau_c^{corr} \, \delta_D^{3+\alpha} \, \theta_s^{2} \left(\frac{\tau_c^{corr}}{\sigma_T}\right) B_{corr}^{1+\alpha} \, \nu_c^{-\alpha}$$

$$\propto \left(\tau_c^{corr}\right)^2 \delta_D^{3+\alpha} \, \theta_s^{2} \, B_{corr}^{1+\alpha} \, \nu_c^{-\alpha}$$

$$\propto \left(\tau_c^{corr}\right)^2 B_{corr}^{1+\alpha} \, \delta_D^{3+\alpha} \, \theta_s^{2} \, \nu_c^{-\alpha}$$

$$\Rightarrow \left(\tau_c^{corr}\right) = \delta_D^{-3-\alpha} \frac{F_t \nu_t^{\alpha}}{\theta_s^2 B_{corr}^{1+\alpha}}$$

$$\Rightarrow \left(\tau_c^{corr}\right)^2 = \delta_D^{-6-2\alpha} \frac{F_t^2 \nu_t^{2\alpha}}{\theta_s^4 (B_{corr}^{1+\alpha})^2}$$

Then:

$$F_{ssc}(\nu_c) \propto \delta_D^{3+\alpha} \left(\tau_{corr}\right)^2 B_{corr}^{1+\alpha} \, \theta_s^{2} \, \nu_c^{-\alpha}$$

$$\propto \delta_D^{3+\alpha} \left[\delta_D^{-6-2\alpha} \frac{F_t^2 \nu_t^{2\alpha}}{\theta_s^4 (B_{corr}^{1+\alpha})^2}\right] B_{corr}^{1+\alpha} \, \theta_s^{2} \, \nu_c^{-\alpha}$$

$$\propto \delta_D^{-3-\alpha} \frac{F_t^2 \nu_t^{2\alpha}}{\theta_s^2 B_{corr}^{1+\alpha}} \, \nu_c^{-\alpha}$$

$$\propto \delta_D^{-3-\alpha} \frac{F_t^2 \nu_t^{2\alpha}}{\theta_s^2 \Big[\underbrace{\delta_D \frac{\theta_s^4 \nu_t^5}{F_t^2}}_{B_{corr}}\Big]^{1+\alpha}} \, \nu^{-\alpha}$$

$$F_{ssc}(\nu_c) \propto \delta_D^{-3-\alpha} \frac{F_t^{2} \nu_t^{2\alpha}}{\theta_s^{2}} \left[\delta_D\left(\frac{\theta_s^{4}\nu_t^{5}}{F_t^{2}}\right)\right]^{-(1+\alpha)} \nu_c^{-\alpha}$$

$$\propto \left(\delta_D^{-3-\alpha}\, \delta_D^{-(1+\alpha)}\right)\left(\theta_s^{-2}\, \theta_s^{-4(1+\alpha)}\right)\left(F_t^{2}\, F_t^{2(1+\alpha)}\right) \left(\nu_t^{2\alpha}\, \nu_t^{-5(1+\alpha)}\right) \nu_c^{-\alpha}$$

$$\propto \delta_D^{-4-2\alpha}\, \theta_s^{-2-4-4\alpha}\, F_t^{2+2+2\alpha}\, \nu_t^{2\alpha-5-5\alpha}\, \nu_c^{-\alpha}$$

$$\propto \delta_D^{-2(2+\alpha)}\, \theta_s^{-6-4\alpha}\, F_t^{4+2\alpha}\, \nu_t^{-3\alpha-5}\, \nu_c^{-\alpha}$$

$$\propto \delta_D^{-2(2+\alpha)}\, \theta_s^{-2(3+2\alpha)}\, F_t^{2(2+\alpha)}\, \nu_t^{-(5+3\alpha)}\, \nu_c^{-\alpha}$$

This is the predicted SSC flux at ν_c which can be compared with observed flux. This allows estimate of Doppler factor in the source. All the parameters $F_{ssc}(\nu_c)$, F_t, ν_t and ν_c relate to observed quantities. The Doppler factor has to match prediction to observation.