Comptonization Spectra

The growth or evolution of a Compton spectrum is determined by the Kompaneets eqn.

$$\frac{\partial n}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n^2 + n\right)\right] \qquad x = \frac{h\nu}{kT}$$

The LHS is

$$\frac{\partial n}{\partial y} = \frac{m_e c^2}{kT_e} \frac{1}{n_e \sigma_T c} \frac{\partial n}{\partial t}$$

$$\Rightarrow \quad \frac{\partial}{\partial y} = \frac{m_e c^2}{kT} \frac{1}{n_e \sigma_T c} \frac{\partial}{\partial t}$$

$$\Rightarrow \quad dy = \frac{kT_e}{m_e c^2} n_e \sigma_T c \, dt$$

$$= \frac{kT_e}{m_e c^2} n_e \sigma_T \, c\,dt$$

$$= \frac{kT_e}{m_e c^2} n_e \sigma_T \, dl$$

The Syunyaev - Zeldovich effect

The S-Z effect is the distortion of the CM background as a result of the IC scattering of hot electrons in the halos of galaxy clusters. We can calculate this effect by using the Kompaneets eqn. Because $T_{gas}(\text{cluster}) \gg T_{rad}(\text{CMB})$ the $\frac{\partial n}{\partial x}$ term in the Kompaneets eqn dominates. This is because

$$\left\{\begin{matrix}\text{Dimensional} \\ \text{analysis}\end{matrix}\right\} \qquad \frac{\partial n}{\partial x} \sim \frac{n}{x} \gg n \quad \text{for} \quad x = \frac{h\nu}{kT} \ll 1$$

Then

$$\frac{\partial n}{\partial y} \simeq \frac{1}{x^2} \frac{\partial}{\partial x} \left(x^4 \frac{\partial n}{\partial x} \right]$$

To quantify the perturbation on the CMB photon density as a result of IC scattering, we write

$$\delta n = \left[\frac{\partial n}{\partial y}\right] dy$$

For a cluster

$$y = \int dy$$

$$= \int \frac{kT_e}{m_e c^2} \underset{\uparrow \text{low}}{n_e} \sigma_T \, dl \qquad \ll 1$$

$$y \to dy$$

$$\delta n \simeq \frac{\partial n}{\partial y} y = y \left[\frac{1}{x^2} \frac{\partial}{\partial x} \left(x^4 \frac{\partial n}{\partial x} \right)\right]$$

Therefore

$$\frac{\delta n}{n} = \frac{y}{n}\frac{\partial n}{\partial y}$$

$$= \frac{y}{n}\left[\frac{1}{x^2}\frac{d}{dx}\left(x^4\frac{dn}{dx}\right)\right]$$

For a Planck spectrum (CMB)

$$n = \frac{1}{e^x - 1} \qquad x = \frac{h\nu}{kT}$$

$$\frac{\delta n}{n} = y\left[\frac{x e^x}{e^x - 1}\right]\left[x \coth\left(\frac{x}{2}\right) - 4\right] \qquad \text{[verify it]}$$

In the R-J limit $x = \frac{h\nu}{kT} \ll 1$

Show that:

$$\frac{\delta n}{n} = -2y \quad \text{[with]} \quad \boxed{y = \int \frac{kT_e}{m_e c^2} n_e \sigma_T \, dl}$$

Hence CMB photons upon interaction with cluster electrons will be pumped up in energy through IC scattering. The resultant effect

$$\left(\frac{\delta n}{n}\right)_{clus} = -2y \quad \Rightarrow$$ [This depletion of CMB photons is then added to CMB at higher frequencies.]

For a BB spectrum

$$n_{ph} = 0.24\left(\frac{k}{\hbar c}\right)^3 T^3$$

$$\delta n_{ph} \approx 0.24\left(\frac{k}{\hbar c}\right)^3 3T^2 \, \delta T$$

Therefore

$$\frac{\delta n_{ph}}{n_{ph}} = \frac{3\ \delta T_{bb}}{T_{bb}} \propto \frac{\delta T_{bb}}{T_{bb}}$$

Hence

$$\frac{\delta T_{bb}}{T_{bb}} \approx -2 \int \frac{kT_e}{m_e c^2} n_e \sigma_T \, dl$$

One also know that a perturbation in the temperature of the CMB is going to result in a perturbation BB intensity. In R-J limit

$$I_\nu = \frac{2\nu^2}{c^2} T_b$$

$$\delta I_\nu = \frac{2\nu^2}{c^2} \delta T_b$$

$$\Rightarrow \delta T_{bb} = \frac{c^2}{2\nu^2} \delta I_\nu$$

$$\Rightarrow \frac{\delta T_{bb}}{T_{bb}} \propto \frac{\delta I_\nu}{I_\nu}$$ [can be verified observationally]

<u>In R-J limit:</u>

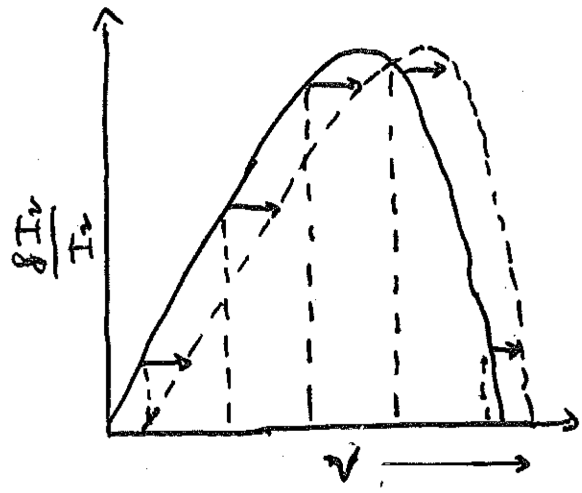
$$\frac{\delta T_{bb}}{T_{bb}} = -\frac{2 k_B}{m_e c^2} \sigma_T \int n_e T_e \, dl \quad \left(\propto \frac{\delta I_\nu (R\text{-}J)}{I_\nu (R\text{-}J)}\right)$$

For clusters : $\boxed{n_e \sim 10^{-2}\ \mathrm{cm}^{-3} \text{ and } L \sim \mathrm{Mpc} \quad \frac{kT_e}{m_e c^2} \sim \frac{1}{50} \text{ with } kT_e \sim 10\,\mathrm{keV}}$

In R-J limit : $$\frac{\delta T_{bb}}{T_{bb}} \approx -8 \times 10^{-4} \qquad T_{bb} \sim 3\,\mathrm{K}$$

$$\delta T_{bb} \approx -10^{-3}\ \mathrm{K}$$ [Significant depletion of CMB]

Longair:



This is the result due to the IC scattering of the CMB photons off $\varepsilon_e \sim$ few keV electrons in clusters. IC drains photons from the CMB spectrum and duplicates the spectrum at higher frequencies.