Dispersion of pulsar signals

Fig. 15.20 of the text book illustrates how the pulsed radio signals of a pulsar are being dispersed as they travel through the interstellar medium (ISM). This means that signals emitted at different photon frequencies will arrive at the observer at slightly different times because of the effect of the ISM plasma. In order to explain this, recall that electromagnetic radiation traveling through a plasma, will do so at a speed which is different from the speed of light in vacuum, c. Specifically, any signal (e.g. the pulse from a pulsar) will travel with the group velocity $v_g = n\,c$, where n is the index of refraction of the plasma, given by

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \qquad (1).$$

Here, $\omega = 2\pi\,\nu$ is the radio frequency of the radiation, and

$$\omega_p = \sqrt{\frac{4\pi\,n_e\,e^2}{m_e}} = 5.63 \times 10^4 \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{1/2} \mathrm{s}^{-1} \qquad (2)$$

is the plasma frequency.

Now, assume that we are observing a pulsed signal from a pulsar at a distance d from Earth at a frequency $\omega \gg \omega_p$. The signal will arrive at Earth at a time

$$t = \int_0^d \frac{ds}{v_g} = \frac{1}{c} \int_0^d \frac{ds}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} \approx \frac{1}{c} \int_0^d ds \left(1 + \frac{1}{2}\frac{\omega_p^2}{\omega^2}\right). \qquad (3)$$

If the signal is observed at two slightly different frequencies, ω_1 and $\omega_2 = \omega_1 + \delta\omega$, with $\delta\omega \ll \omega_1$, then the difference in arrival time between the signals at the two different photon frequencies will be

$$\delta t \approx \frac{1}{2c} \int_0^d ds \left(\frac{\omega_p^2}{\omega_1^2} - \frac{\omega_p^2}{\omega_2^2}\right) \approx \frac{1}{c} \int_0^d ds\, \frac{\omega_p^2}{\omega_1^3}\,\delta\omega = \frac{4\pi\,e^2}{m_e c\,\omega_1^3}\,\delta\omega \int_0^d n_e(s)\,ds. \qquad (4)$$

Thus, the arrival times of the pulses at different photon energies are of the form $\delta t = c_1\,DM\,\delta\omega$, where the quantity

$$DM = \int_0^d n_e(s)\,ds \qquad (5)$$

is called the **dispersion measure**. Using pulsars as extremely precise clocks, Eq. (4) allows radio observers to measure the dispersion measure and thus the average electron density in clouds in the ISM through which a pulsar signal is traveling towards us.